DISCRETE MATHEMETICS MID-TERM EXAM

2010/11/15

1. [10%] Determine the number of integer solutions to the equation
   $x_1 + x_2 + ... + x_6 + x_7 = 10, \quad 0 \le x_i, \quad 1 \le i \le 6, \quad 0 < x_7$.
2. [10%] For any universe $\mathcal{U}$ and any sets $A, B \subseteq \mathcal{U}$, which following statements are equivalent and explain why.
   (a) $\overline{A} \subseteq B$ (b) $A \bigcup B = B$ (c) $A \bigcap B = A$
3. [10%] For A = {1, 2, 3, 6}.
   (a) How many relations on A are antisymmetric ?
   (b) If $|A| = n > 0$, how many relations on A are antisymmetric ?
4. [10%] Let $M = (S, \varphi, o, \nu, \omega)$ be a finite state machine where $S = \{s_0, s_1\}$
   $\varphi = \{00, 01, 10, 11\}$, $o = \{0,1\}$, and ν, ω are determined by Table 1.

   Table 1:

	ν				ω			
	00	01	10	11	00	01	10	11
s_0	s_0	s_0	s_0	s_1	0	1	1	0
s_1	s_0	s_1	s_1	s_1	1	0	0	1

   Draw the state diagram for this finite state machine.

5. [10%] Let $|A| = 6$.
   (a) How many closed binary operations are there on A?
   (b) How many of these closed binary operations are commutative?
6. [10%] Determine the greatest common divisor of 231 and 1820 and express the result as a linear combination of these integers. (Use the Euclidean Algorithm)
7. [10%] For each of the following functions g: $\mathbf{R} \rightarrow \mathbf{R}$, determine whether the function is one-to-one and whether it is onto. If the function is not onto, determine the range $g(\mathbf{R})$. (a) $g(x) = x + 7$ (b) $g(x) = x^2 + x$
8. [10%] For each of the following statements about relations on a set A, where $|A| = n$, determine whether the statement is true or false. If it is false, give a counterexample.
   (a) R_1, R_2 are relations on A , $R_2 \supseteq R_1$, and R_2 is antisymmetric. $\Rightarrow$ R_1 is antisymmetric.
   (b) R_1, R_2 are relations on A, $R_2 \supseteq R_1$, and R_2 is transitive. $\Rightarrow$ R_1 is transitive.
9. [10%] Let S = {3, 7, 11, 15, 19,…, 95, 99, 103}. How many elements must we select from S to insure that there will be at least two whose sum is 110?
10. [10%] An auditorium has a seating capacity of 800. How many seats must be occupied to guarantee that at least two people seated in the auditorium have the same first and last initials?