DISCRETE MATHEMATICS
Final Examination (2011/01/10)

1. [10%] If $A = \{w, x, y, z\}$, determine the number of relations on A that are (a) symmetric and contain (x,y) (b) symmetric and antisymmetric.
2. [10%] Determine the coefficient of x^8 in $\dfrac{1}{(x-3)(x-2)^2}$.
3. [10%] In each of the following, the function $f(x)$ is the exponential generating function for the sequence $a_0, a_1, a_2, ...,$ whereas the function $g(x)$ is the exponential generating function for the sequence $b_0, b_1, ...$ Express $g(x)$ in terms of $f(x)$ if

   $a_n = 5^n, n \in \mathbf{N},$

   $b_3 = -1,\ \ b_n = a_n, n \in \mathrm{N}, n \neq 3.$
4. [10%] Solve the recurrence relation
   $a_{n+2} - 4a_{n+1} + 3a_n = -200,\ \ n \geq 0,\ \ a_0 = 3000,\ \ a_1 = 3300.$
5. [10%] Solve the recurrence relation
   $a_n = 2(a_{n-1} - a_{n-2})$, where $n \geq 2$ and $a_0 = 1,\ a_1 = 2.$
6. [10%] (a) How many different paths of length 2 are there in the undirected graph G in Fig. 1? (b) Let $G = (V, E)$ be a loop-free undirected graph, where $V = \{v_1, v_2, ..., v_n\}$ and $\deg(v_i) = d_i$, for all $1 \leq i \leq n$. How many different paths of length 2 are there in G?

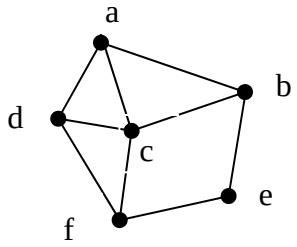

Fig. 1

7. [10%] Find all the non-isomorphic complete bipartite graphs $G = (V, E)$, where $|V| = 6$.
8. [10%] Find and solve a recurrence relation for the number of ways to park motorcycles and compact cars in a row of n spaces if each cycle requires one space and each compact needs two. (All cycles are identical in appearance, as are the cars, and we want to use up all the n spaces.)
9. [10%] In how many ways can 3000 identical envelopes be divided, in packages of 25, among four student groups so that each group gets at least 150, but not more than 1000, of the envelopes?
10. [10%] How many 6×6 (0, 1)-matrices A are there with $A = A^{tr}$?