CALCULUS - *for Computer Science and Engineering*

Midterm EXAM

Department of Computer Science and Engineering
National Sun Yat-sen University

November 24, 2010, 13:15~ 16:00

NAME: _____ Student ID Nr.: _____

Instructor: _____

General Instructions:

1. Do not open this exam until you are told to begin.
2. This exam has 8 pages including this cover. There are 5 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to show how you arrived at your solution.
8. Please turn off all cell phones and pagers and remove all headphones.

Problem	1	2	3	4	5	6	7	Total
Points	48	10	10	10	10	10	10	108
Score								

1. Please fill in the following blanks with your answers. *(4% for each)*

(1) Find the simpler expression for $e^{\ln(x^2+y^2)}$ = _____

(2) $\lim_{x\to 0}\dfrac{\sin(\sin x)}{x}=$ _____

(3) If $\lim_{x\to -2}\dfrac{f(x)}{x^2}=1$, then $\lim_{x\to -2} f(x)=$ _____

(4) $f(x)=\begin{cases} x^2-1, & x<3 \\ 2ax, & x\geq 3 \end{cases}$ is continuous at every x if a = _____

(5) $y=x^2\sin x+2x\cos x-2\sin x$, $\dfrac{dy}{dx}=$ _____

(6) $y=xe^{-x}+e^{3x}$, $\dfrac{dy}{dx}=$ _____

(7) $2x^3-3y^2=8$, $\dfrac{d^2y}{dx^2}$ (in terms of x and y) = _____

(8) $\lim_{x\to 0}\dfrac{\sin 5x}{\sin 4x}=$ _____

(9) $\lim_{x\to 9}\dfrac{\sqrt{x}-3}{x-9}=$ _____

(10) $x^y=y^x$, $\dfrac{dy}{dx}=$ _____

(11) $\lim_{x\to\infty}\dfrac{x-2x^2}{3x^2+5x}=$ _____

(12) $y=(1-x)(1+x^2)^{-1}$, $y'=$ _____

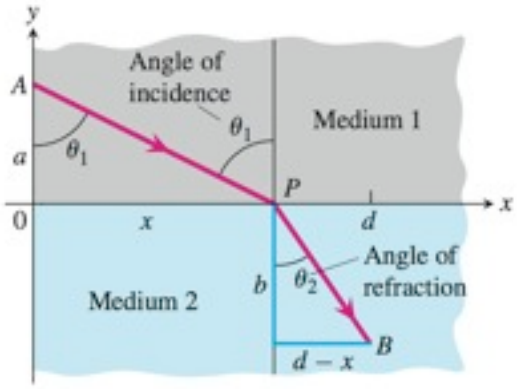
2. (10%) Prove that $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$ (θ in radians)

3. (10%) Prove that $e = \lim_{x \to 0}(1 + x)^{1/x}$

4. (10%) Find the derivative $f'(x)$ if $f(x) = x^x$, $x > 0$.

5. (10%) Fermat's Principle and Snell's Law

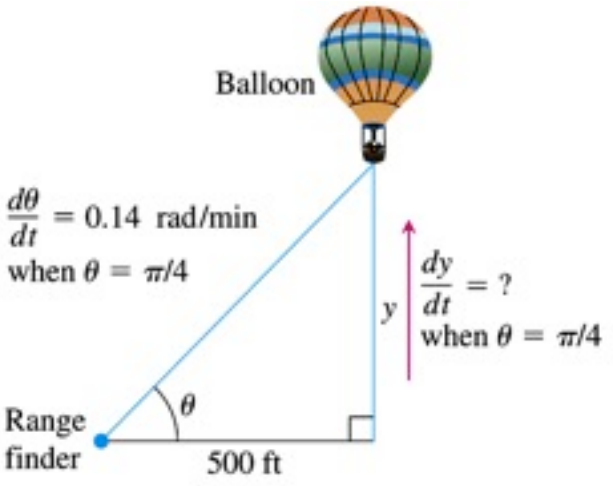
The speed of light depends on the medium through which it travels, and is generally slower in denser medium. Fermat's principle in optics states that light travels from one point to another along a path for which the time of travel is a minimum. Find the path that a ray of light will follow in going from a point A in a medium where the speed of light is c_1 to a point B in a second medium where its speed is c_2.



Answer: _____

6. (10%) Sketch the graph of $f(x) = \dfrac{(x+1)^2}{1+x^2}$. Find the horizontal asymptote, points of inflection, local minimum and local maximum of f(x) and label them in your graph.

7. (10%) A hot air balloon rising straight up from a level field is tracked by a range finder 500 ft from the liftoff point. At the moment the range finder's elevation angle is $\pi/4$, the angle is increasing at the rate of 0.14 rad/min. How fast is the balloon rising at that moment?



(End of this exam.)