

Qualifying Exam: Probability

1. (20%) A test for a certain rare disease is assumed to be correct 95% of the time. In a population, the probability of a person having the disease is 0.001. Suppose a person chosen at random test positive for the disease. What is the probability that he does have the disease?
2. (20%) A person is given two questions and must decide which one to answer first. Question 1 will be answered correctly with probability 0.8 and prize money 100, while Question 2 will be answered correctly with probability 0.5 and prize money 200. If the first attempted question is answered correctly, the person is allowed to answer the other question. Prize money can be accumulated. What is the expected prize money of answering Question 1 first? Question 2 first?
3. (20%) Let the joint probability density of X and Y be

$$f(x, y) = \begin{cases} 2xy, & 0 \leq y \leq 2x \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$

What is $E[X|Y]$?

4. (20%) Two continuous random variables X and Y are independent and uniformly distributed in $[0, 1]$. What is the probability density function for $Z = X + Y$?
5. (20%) Jack has \$600. In one game, he wins \$100 with probability 0.4, or loses \$100 with probability 0.6. Different games are assumed to be independent. He continues to gamble until either he loses all his money or he holds \$1,000. What is the probability that he loses?