Qualifying Exam: Probability

1. (20%) A 2-state Markov chain has the following transition probability matrix

$$\mathbf{P} = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}$$

What is the steady-state probability for the first state?

2. (20%) Suppose N is a geometric random variable with mean 3. What is the distribution of

$$Y = \sum_{i=1}^{N} X_i,$$

where the random variables X_i's are i.i.d. exponential random variable with mean 1?

3. (20%) A stick of length l is broken, and the break point is uniformly distributed on $[0, l]$. The part containing the left end is broken again, and the break point is again uniformly distributed. Let the length of the remaining part containing the left end be X. What is the variance of X?

4. (20%) Two continuous random variables X and Y are independent and uniformly distributed on $[0, 1]$. What is the probability density function for $Z = \frac{X}{Y}$?

5. (20%) Let $Z = \begin{bmatrix} X & Y \end{bmatrix}^T$ be a Gaussian random vector with mean $\begin{bmatrix} \mu_x & \mu_y \end{bmatrix}^T$ and variance $\begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix}$. Derive the conditional distribution $p_{X|Y=y}(x)$.