Dept. of Computer Science and Engineering, National Sun Yat-sen Univ.
Second Semester of 2008 PhD Qualifying Exam Computer Algorithms

1. Explain the following terms. (15%)
   (a) *NP, NP-complete*
   (b) *node cover problem*
   (c) *amortized time analysis*

2. In the solution searching strategy, there are two basic ways to visit solution nodes: *depth-first search* and *breadth-first search*.
   (a) What data structure should be used in depth-first search? Why? (5%)
   (b) What data structure should be used in breadth-first search? Why? (5%)
   (c) What is the *best-first search* scheme? (5%)

3. (a) Give an example to explain how the *straight insertion sort* method works. (5%)
   (b) Analyze the number of data exchanges in the average case if there are n elements to be sorted. (10%)

4. (a) The *selection* problem is to select the kth smallest element among n input elements. Please design an algorithm to solve the problem. The time complexity of your algorithm should not be more than $O(n)$. (Hints: The *prune-and-search* strategy is a good approach.) (10%)
   (b) What is the general recurrence form for computing the time complexity of a prune-and-search algorithm (not particular for the selection problem)? (5%)

5. (a) Explain the *longest common subsequence* (LCS) problem. And, then give an example to illustrate your answer. Note that you should give both explanation and example. (5%)
   (b) In the *edit distance* (ED)problem, suppose the following operations are allowed: (1) match with cost 0; (2) insertion with cost 1; and (3) deletion with cost 1. Given two sequences with lenghts n and m, the ED problem is to calculate the minimum cost, denoted as D, for transforming one sequence into the other. Suppose the LCS length of these two sequences is L. Please derive the relationship (formula) between D and L. Do not write programs or algorithms. (10%)

6. Suppose *Swap(x,y)* denotes the operation of swapping the contents of variables x and y, which can be implemented as follow:

   $temp = x$; // temp is temporary storage
   $x = y$;
   $y = temp$;

   Now suppose we have to do the following three swapping operations:

   $Swap(b,c)$;
   $Swap(a,b)$;
   $Swap(c,d)$;

   One can easily see that the straightforward implementation will require $3 \times 3 = 9$ assignment (movement) operations. How do you rewrite the above three swapping operations directly by using 5 assignment operations (not 9)? (10%)

7. Given a set S of n positive integers, the 4-subset problem is to determine whether there exist four distinct elements a, b, c and d in S such that $a+b+c=d$. Please design an algorithm to solve this problem, and analyze the time complexity of your algorithm. (15%)

   (a) If the time complexity of your algorithm is $O(n^3 \log n)$, then you will get only 5% points.

   (b) If the time complexity of your algorithm is $O(n^2 \log n)$, then you will get the full 15% points.