B3043002 ALGORITHMS

Department of Computer Science and Engineering
close-book final exam

You may answer the questions in any order. Unless the details are requested, you may directly use anything that we have shown in class in a "black-box" way. Notice that dishonest behaviors and attempts will be punished most seriously. Also, the answer given by the "eye survey" will get zero grade.

1. (20%) **True/False**. To get credit, you must give reasons for your answers !!
   (a) We can solve all problems by non-deterministic algorithms.
   (b) Is the decision version of *Independent Set Problem* in **NP**?
   (c) Let P be a shortest path from node s to node t in a graph G. If the weight of each edge in G is increased by one, then P remains a shortest path from s to t.
   (d) If the graph G has a cycle with a unique heaviest edge e, then e cannot be part of any minimum spanning tree of G.
   (e) The shortest path between two nodes is necessarily part of some minimum spanning tree.

2. (10%) A DNA sequence is composed of A, T, C, and G. Please design an encoding table for the sequence **ATACCGA** such that the code in the encoding table minimizes the length of the sequence. (Note that your encoding scheme has to be lossless.)

3. (10%) Consider the flow network in Figure 1. Find the maximum flow and show a minimum cut.

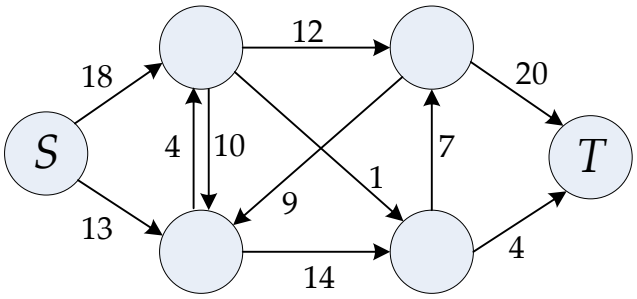

Figure 1

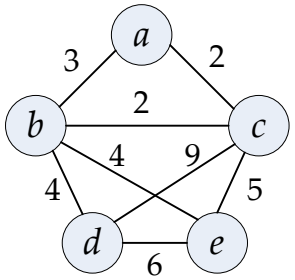

Figure 2

Figure 1: test

4. (10%) Describe how to use *Kruskal's algorithm* to find a minimum spanning tree of the graph shown in Figure 2.

5. (15%) In Figure 3, use *Dijkstra's algorithm* to find all shortest paths from vertex a to other vertices, and analyze its time complexity. Under what condition *Dijkstra's algorithm* will not work? Give an example to explain your answer.

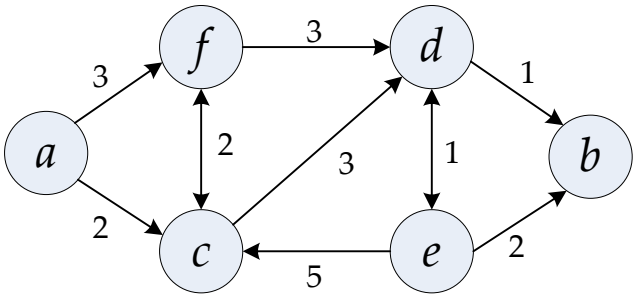

Figure 3

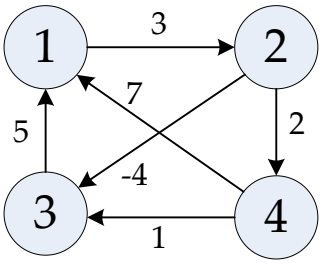

Figure 4

6. (20%) We want to find all-pairs shortest paths in the graph of Figure 4. Please answer the following questions:

   (a) Suppose the *Floyd-Warshall algorithm* is used. Please write down the recurrence relation, and compute the intermediate results (distance matrices) of d_0, d_1 and d_2 for all pairs of nodes.

   (b) Describe how to use *Dijkstra's algorithm* to find all-pairs shortest paths of the graph. (Show the re-weighting process.)

7. (10%) Consider the subprograms for the following problems and write down the corresponding recurrence relations:

   (a) Given a sequence S of numbers, an *increasing* subsequence is a subsequence of S in which the numbers are getting strictly larger. Find the *longest increasing subsequence* of S, and show it when $S = 5, 2, 8, 6, 3, 6, 9, 7$.

   (b) Let X and Y be two sequences. A sequence Z is a *supersequence* of X and Y if both of them are subsequence of Z. Find the *shortest common supersequence* of X and Y.

8. (5%) A *bottleneck spanning tree* T of an undirected graph G is a spanning tree of G whose largest edge weight is minimum over all spanning trees of G. We say that the value of the bottleneck spanning tree is the weight of the maximum-weight edge in T. Argue that a minimum spanning tree is also a bottleneck spanning tree.