Linear Algebra Final 2010.1.13

1. (20%) Given a matrix
$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix},$$
find three eigenvectors of A that are linearly independent. (hint: characteristic equation, homogeneous solution)

2. (20%) Find the symmetric factorization $A = LDL^T$, where
$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 6 & 4 \\ 0 & 4 & 11 \end{bmatrix}.$$
Note that L is lower-triangular, D is diagonal. (hint: $A = LU$ first)

3. (20%) Compute the determinant of the Hadamard matrix H, where
$$H = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}.$$

4. (20%) Construct the projection matrix P onto the subspace in R^3 spanned by $(1, 1, 1)$ and $(0, 1, 3)$.

5. (40%) Find the singular value decomposition of $A = U\Sigma V^T$, where
$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$