DISCRETE MATHEMETICS MID-TERM EXAM

2009/11/18

1. [10%] An executive buys \$2490 worth of presents for the children of her employees. For each girl she gets an art kit costing \$33; each boy receives a set of tools costing \$29. How many presents of each type did she buy?
2. Let A be a set with $|A| = n$, and let R be a relation on A that is antisymmetric. (a) [5%] What is the maximum value for $|R|$? (b) [5%] How many antisymmetric relations can have this size?
3. For each of the following relations, determine whether the relation is reflexive, symmetric, antisymmetric, or transitive.
   (a) [2%] $R \subseteq Z^+ \times Z^+$ where $a\ R\ b$ if $a|b$ (read "a divides b," as defined in Section 4.3).
   (b) [2%] R is the relation on Z where $a\ R\ b$ if $a|b$.
   (c) [2%] On the set A of all lines in R^2, define the relation R for two lines l_1, l_2 by $l_1\ R\ l_2$ if l_1 is perpendicular to l_2.
   (d) [2%] R is the relation on Z where $x\ R\ y$ if $x + y$ is odd.
   (e) [2%] R is the relation on $Z \times Z$ where $(a,b)\ R\ (c, d)$ if $a \le c$. [Note: $R \subseteq (Z \times Z) \times (Z \times Z)$]
4. Let $A = \{1, 2, 3, 4, 5, 6, 7\}$ and $B = \{v, w, x, y, z\}$. Determine the number of functions $f : A \to B$ where (a) [5%] $f(A) = \{v, x\}$; (b) [5%] $|f(A)| = 2$.
5. How many times must we roll a single die in order to get the same score (a) [5%] at least twice? (b) [5%] at least n times, for $n \ge 4$? (Use the Pigeonhole Principle)
6. [10%] Find the coefficient of x^{50} in $(x^7 + x^8 + x^9 + \ldots)^6$.
7. [10%] Solve the recurrence relation: $2a_{n+3} = a_{n+2} + 2a_{n+1} - a_n$, $n \ge 0$, $a_0 = 0$, $a_1 = 1$, $a_2 = 2$.
8. A ship carries 48 flags, 12 each of the colors red, white, blue, and black. Twelve of these flags are placed on a vertical pole in order to communicate a signal to other ships. (Use the exponential generating function)
   (a) [10%] How many of these signals use an even number of blue flags and an odd number of black flags?
   (b) [10%] How many of the signals have at least three white flags or no white flags at all?
9. Let A be a set with $|A| = n$.
   (a) [5%] How many binary relations on A are reflexive but not symmetric?
   (b) [5%] How many binary relations on A are neither reflexive nor symmetric?