DISCRETE MATHEMATICS

Final Examination (2010/01/13)

1. [10%] Solve the following recurrence relation. (No final answer should involve complex numbers.) $a_n + 2a_{n-1} + 2a_{n-2} = 0$, $n \geq 2$, $a_0 = 1$, $a_1 = 3$

2. [10%] Solve the following recurrence relation. $a_{n+2}^2 - 5a_{n+1}^2 + 6a_n^2 = 7n$,

   $n \geq 0, a_0 = a_1 = 1$

3. [10%] (If two subgraphs are isomorphic but have different vertex sets, consider them distinct.) (a) How many subgraphs $H = (V, E)$ of K_6 satisfy $|V| = 4$?
   (b) For $n \geq 3$, how many subgraphs does K_n have?

4. [10%] Find the maximum length of a circuit in

   (a) K_{10} (b) K_{2n}, $n \in Z^+$

5. [10%] Determine whether or not each of the following sets of numbers is a ring under ordinary addition and multiplication. If not, explain why.
   (a) $R =$ the set of positive integers and zero
   (b) $R = \{a + b\sqrt{2} + c\sqrt{3} \mid a \in Z;\ b, c \in Q\}$

6. [10%] Find a simultaneous solution for the system of three congruences: (use "Chinese Remainder Theorem")
   $X \equiv 3 \pmod{17}$, $X \equiv 10 \pmod{16}$, $X \equiv 0 \pmod{15}$

7. [10%] How many units are there in Z_{72}?

8. [10%] Let $(F, +, \circ)$ be a field where F is a nonempty set and "+", "$\circ$" are two binary operations on F. Please describe the eleven requirements the field must satisfy.

9. [10%] Please show that the multiplicative group ($U_9 = \{1, 2, 4, 5, 7, 8\}, \circ$) is cyclic, where $x \circ y = xy \bmod 9$ for each x, y in U_9, for example: $4 \circ 5 = 20 \bmod 9 = 2$.

10. [10%] Let $(G, \circ)$ and $(H, *)$ be two groups. Prove that if f: $G \rightarrow H$ is a homomorphism, then $f(S)$ is a subgroup of H for each subgroup S of G.