Linear Algebra Midterm

1. (10%) Prove the triangular inequality: for any two vectors $x, y \in R^n$,

$$|x + y| \leq |x| + |y|.$$

2. (10%) A unit upper-triangular matrix is an upper-triangular matrix with all diagonal entries being 1. Prove that the product of two unit upper-triangular matrices is unit upper-triangular.

3. (10%) Let $a = [4 \ 0 \ -3]^T, b = [3 \ 1 \ -7]^T$. Find the projection vector of b on a.

4. (10%) Given a square matrix

$$A = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix},$$

find matrices B, C such that

$$\begin{cases} B + C = A \\ B - B^T = 0 \\ C + C^T = 0. \end{cases}$$

5. (10%) Given

$$M_1 = \begin{bmatrix} -2 & 1 \\ 0 & 5 \\ 4 & -3 \end{bmatrix}, M_2 = \begin{bmatrix} 1 & -6 \\ 0 & 2 \end{bmatrix}, M_3 = \begin{bmatrix} -4 & 2 & 1 \end{bmatrix}, M_4 = \begin{bmatrix} 7 \\ -1 \\ 5 \end{bmatrix},$$

find the set of matrices

$$S = \{N \mid N = M_i M_j \text{ for some } i \neq j\}.$$

6. (10%) Solve the following system of linear equations

$$\begin{aligned} x - \quad y - \quad 3z &= 8 \\ 4x - \quad 2y - \quad 6z &= 19 \\ 3x - \quad 6y - \quad 17z &= 41. \end{aligned}$$

7. (20%) Solve the following homogeneous system of linear equations

$$
\begin{aligned}
u - 3v + 2w - 4x + 8y + 17z &= 0 \\
3u - 9v + 6w - 12x + 24y + 49z &= 0 \\
-2u + 6v - 5w + 11x - 18y - 40z &= 0.
\end{aligned}
$$

8. (20%) Let

$$
A = \begin{bmatrix} 2 & 1 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ 0 & -2 & 1 & 0 \end{bmatrix} .
$$

(a) Find an orthonormal basis for the row space of A.

(b) Find a basis for the orthogonal complement of the above space.