Linear Algebra Final Exam 0930-1130 2009/01/13

1. (10) Let
$$A = \begin{bmatrix} -1 & 3 & 2 \\ 1 & 2 & -3 \\ 2 & 1 & -2 \end{bmatrix}, \quad u = \begin{bmatrix} 1 \\ -9 \\ -3 \end{bmatrix}.$$
Equate u as a linear combination of the column vectors of A.

2. (10) Find the cofactor matrix of A, where
$$A = \begin{bmatrix} -1 & 3 & 2 \\ 1 & 2 & -3 \\ 2 & 1 & -2 \end{bmatrix}.$$
then solve the equation $Ax = b$ where $b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

3. (15) Find the inverse of A, where
$$A = \begin{bmatrix} -1 & 3 & 2 & 1 \\ 1 & 2 & -3 & 1 \\ 2 & 1 & -2 & 1 \\ 1 & 1 & -2 & 1 \end{bmatrix}.$$

4. (10) A linear transform T maps $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ to $\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ to $\begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}$. What is the result of T operating on $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$?

5. (10) Compute the determinant of $Z = XYX^{-1}$, where
$$X = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix}, \quad Y = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 4 \\ 2 & 0 & 1 \end{bmatrix}.$$

6. (10) Compute the singular value decomposition of A, where

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}.$$

7. (20) Let

$$H = \begin{bmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix}.$$

Represent e^H as a matrix.

8. (15) Let

$$A = \begin{bmatrix} 1 & -2 & -2 & 0 \\ 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 6 \end{bmatrix}.$$

Find an orthonormal basis for the row space of A.