DISCRETE MATHEMATICS

Midterm Examination

2008/12/27

(Two pages)

1. [10%] An executive buys $2490 worth of presents for the children of her employees. For each girl she gets an art kit costing $33; each boy receives a set of tools costing $29. How many presents of each type did she buy?

2. [10%] Let $A = \{x, a, b, c, d\}$.
   (a) How many closed binary operations f on A satisfy $f(a, b) = c$?
   (b) How many of the functions f in part (a) have an identity?

3. [10%] If G is the directed graph for a relation R on A, with $|A| = n$, and (A, R) is a total order, how many edges (including loops) are there in G?

4. [10%] Let $A = \{1, 2, 3, 4, 5\} \times \{1, 2, 3, 4, 5\}$, and define R on A by $(x_1, y_1)\, R\, (x_2, y_2)$ if $x_1 + y_1 = x_2 + y_2$.
   (a) Verify that R is an equivalence relation on A.
   (b) Determine the equivalence classes [(1, 3)], [(2, 4)], and [(1, 1)].

5. [10%] Determine the sequence generated by each of the following generating functions.
   (a) $f(x) = 1/(1 + 3x)$
   (b) $f(x) = 1/(3 - x)$

6. [10%] Find the coefficient of x^{20} in $(x^2 + x^3 + x^4 + x^5 + x^6)^5$.

7. [10%] A company hires 11 new employees, each of whom is to be assigned to one of four subdivisions. Each subdivision will get at least one new employee. In how many ways can these assignments be made? **Note that** you should use generating or exponential generating functions to solve this problem and show the details of your answer.

8. [10%] Determine the coefficient of x^8 in $\dfrac{1}{(x-3)(x-2)^2}$.

9. [10%] **Prove that** there are infinitely many primes.

10. Let M be the finite state machine given in the state diagram shown below.
    (a) [10%] Minimize machine M.
    (b) [5%] Find a (minimal) distinguishing string for state s_3 and state s_6.