DISCRETE MATHEMATICS

Final Examination (2009/01/15)

1. [10%] Find the unique solution for the following recurrence relation: $3a_{n+1} - 4a_n = 0,\ n \ge 0,\ a_1 = 5$.

2. [10%] Solve the following recurrence relation. (No final answer should involve complex numbers.) $a_n + 2a_{n-1} + 2a_{n-2} = 0,\ n \ge 2, a_0 = 1, a_1 = 3$

3. [10%] Solve the recurrence relation $a_{n+2} - 6a_{n+1} + 9a_n = 3(2^n) + 7(3^n)$ where $n \ge 0$ and $a_0 = 1, a_1 = 4$.

4. [5%] (a) How many subgraphs $H = (V, E)$ of K_6 satisfy $|V|$=4? [5%] (b) For $n \ge 3$, how many subgraphs does K_n have?

5. [10%] If $G = (V, E)$ is a connected graph with $|E|$=17 and $\deg(v) \ge 3$ for all $v \in V$, what is the maximum value for $|V|$?

6. [5%] (a) For $n \ge 3$, how many different Hamilton cycles are there in the complete graph K_n? [5%] (b) Nineteen students in a nursery school play a game each day where they hold hands to form a circle. For how many days can they do this with no student holding hands with the same playmate twice?

7. [10%] Let $(F, +, \circ)$ be a field where F is a nonempty set and "+", "$\circ$" are two binary operations on F. Please describe the eleven requirements the field must satisfy.

8. [10%] Let $(G, \circ)$ be a group where G is a nonempty set and "$\circ$" is a binary operation on G. Please describe the four requirements the group must satisfy.

9. [5%] (a) Find $[15]^{-1}$ in $\mathbf{Z}_{13}$. [5%] (b) Find $[18]^{-1}$ in $\mathbf{Z}_{35}$?

10. [10%] Find an integer m such that $0 < m < 23 \cdot 29 \cdot 31$ and $\begin{cases} m \equiv 0 \pmod{23} \\ m \equiv 1 \pmod{29} \\ m \equiv 2 \pmod{31} \end{cases}$ by the Chinese Remainder Theorem. **(You should show how to get your answer.)**