Calculus *for Computer Science and Engineering*: Midterm

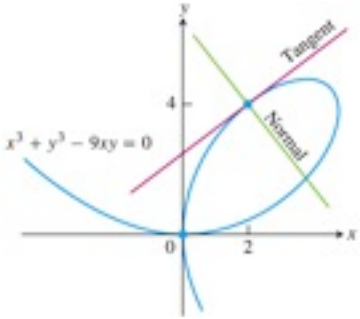
Close Book, total 2 page, 2008.11.26, 13:10~16:00

1. (10%) Prove that $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$ (θ in radians)

2. (10%) If u(x) and v(x) are differentiable, prove that: $\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$.

3. (18%) Find the derivative (dy/dx) of:
   a. $y=\frac{1}{x}(x^2+e^x)$
   b. $y^2=x^2+\sin(xy)$
   c. $y=x^x,\ x>0$
   d. $y=\sin^{-1}x^2$
   e. $y=x(\ln x)^2$
   f. $x^y=y^x$

4. (10%) Show that the point (2,4) lies on the curve $x^3+y^3-9xy=0$. Then find the tangent and normal to the curve there.



5. (10%) Prove that $e^x\ge 1+x$ if $x\ge 0$.
   Use the result to show that $e^x\ge 1+x+\frac{1}{2}x^2$.

6. (10%) Sketching the graph of $y=x^4+2x^3$. Included the coordinates of any local extreme points and inflection points.

7. (6%) Let $f(x)=\begin{cases}3-x, & x<2\\ 2, & x=2\\ x/2, & x>2\end{cases}$
   a. Find $\lim_{x\to 2^+}f(x)$, $\lim_{x\to 2^-}f(x)$, and $f(2)$.
   b. Does $\lim_{x\to 2}f(x)$ exist? If so, what is it? If not, why not?

8. (10%) Find the limit.
   a. $\lim_{x\to\infty}\frac{\sqrt{4x^2+1}}{x+1}$

b. $\lim_{x\to 1} \sin^{-1}\left(\frac{1-\sqrt{x}}{1-x}\right)$

9. (16%) Find the derivative of y with respect to x or θ, as appropriate.

a. $y = \sqrt[3]{\frac{x(x+1)(x-2)}{(x^2+1)(2x+3)}}$

b. $y = \log_5(3x-7)$

c. $y = (\theta^2 + \sec\theta + 1)^3$

d. $y^2 = 2e^{-1/x}$

10. (10%) A hot air balloon rising straight up from a level field is tracked by a range finder 500 ft from the liftoff point. At the moment the range finder's elevation angle is $\pi/4$, the angle is increasing at the rate of 0.14 rad/min. How fast is the balloon rising at that moment?