

**CALCULUS - *for Computer Science and Engineering***  
**FINAL EXAM**

Department of Computer Science and Engineering  
National Sun Yat-sen University

**January 14, 2009, 13:15~ 16:00**

NAME: \_\_\_\_\_ Student ID Nr.: \_\_\_\_\_

Signature: \_\_\_\_\_ Instructor: \_\_\_\_\_

**General Instructions:**

1. Do not open this exam until you are told to begin.
2. This exam has 6 pages including this cover. There are 5 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to show how you arrived at your solution.
8. Please turn off all cell phones and pagers and remove all headphones.

| Problem | 1  | 2  | 3  | 4  | 5  | Total |
|---------|----|----|----|----|----|-------|
| Points  | 36 | 10 | 20 | 20 | 15 | 101   |
| Score   |    |    |    |    |    |       |

1. Circle True or False for each of the following problems. Circle 'True' only if the statement is always true. No explanation is necessary.

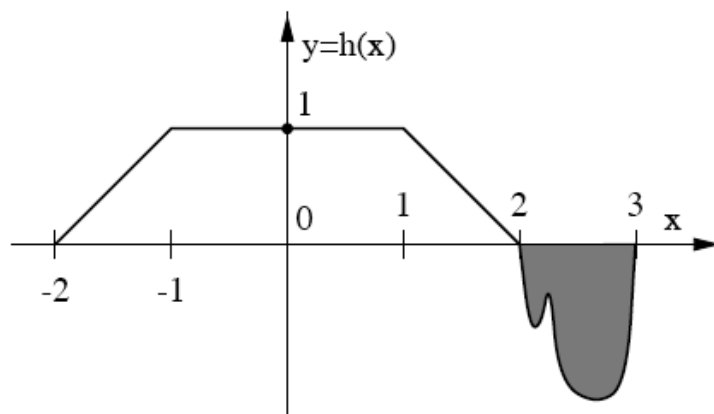
Part I: (2% for each)

- True False** (a)  $\log(10^n) = n$
- True False** (b) if  $7.32 = e^t$ , then  $t = \frac{7.32}{e}$
- True False** (c)  $\ln(A/B) = \ln(A) + \ln(B^{-1})$
- True False** (d)  $x^{10000000}$  will eventually become much larger than  $e^{0.00000001x}$ .
- True False** (e) The polynomial  $p(x) = (x-2)^2(x-5)^4 + 2$  has two roots, a double root at  $x=2$  and a quadruple root at  $x=5$ .
- True False** (f) Every rational function has a vertical asymptote.
- True False** (g) The function  $r(x) = 3x^5 + 2x^3 + 7x + 13$  is a rational function.
- True False** (h) If  $f(x) = \ln(x)$  then  $f^{-1}(x) = \frac{1}{\ln(x)}$ .

Part II: (4% for each)

- True False** (a) Suppose that  $f$  is differentiable and  $f(2) = f(6)$ . Then there must be at least one point  $c \in (2,6)$  with  $f'(c) = 0$ .
- True False** (b) Suppose that  $g$  is continuous,  $g(1) = 5$  and  $g(5) = 10$ . Then the equation  $g(c) = 7$  must have a solution  $c \in (1,5)$ .
- True False** (c) If  $f(x) > x$  for all  $x$ , then  $\int_0^{10} f(x) dx > 5$ .
- True False** (d) If  $f'(x) = g'(x)$  then  $f(x) = g(x)$ .
- True False** (e)  $\int_a^b f(x)g(x) dx = \left(\int_a^b f(x) dx\right)\left(\int_a^b g(x) dx\right)$  for any  $f$  and  $g$ .

2. (10%) Let  $h(x)$  be the function graphed below, and suppose that the shaded region has area equal to 1. Find the average value of  $h(x)$  over the interval  $-2 \leq x \leq 3$ .



**ANS-- The Average Value is:** \_\_\_\_\_

3. (4% for each) Compute:

(a)  $\lim_{x \rightarrow \infty} \frac{x \sin(x)}{\cos(x) - 1} =$  \_\_\_\_\_

(b)  $\int_0^{\frac{\pi}{3}} \sin(x) dx =$  \_\_\_\_\_

(c)  $\frac{d}{dx} \sin(e^x) =$  \_\_\_\_\_

(d)  $\int (x^2 + 3x + 5) dx =$  \_\_\_\_\_

(e)  $\lim_{x \rightarrow \infty} \frac{3x - e^x}{8x^2 + \ln x} =$  \_\_\_\_\_

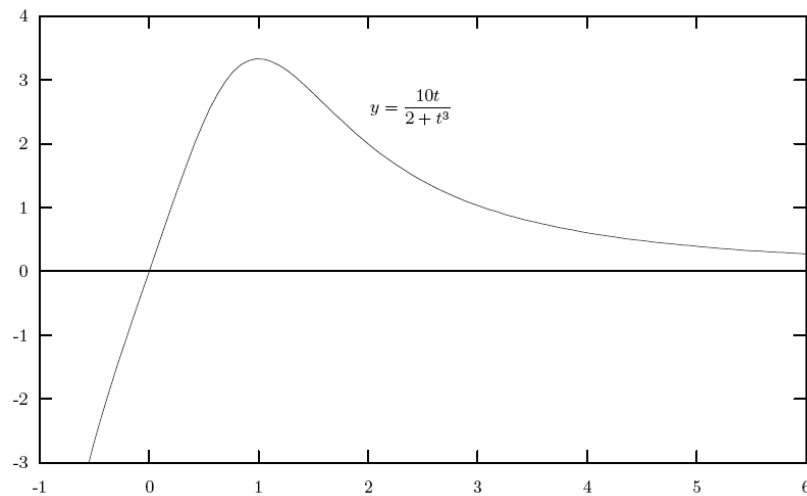
4. (5% for each) Radioactive isotopes are often characterized by their half-life. Fill each of the four blanks (i)–(iv) in the table below. Carefully show step-by-step algebra that justifies your answers to (i) and (iii), as indicated.

| ISOTOPE           | Half-Life (year) | Initial Amount (gram) | Amount after 1000 years (gram) | Amount after 10000 years (gram) |
|-------------------|------------------|-----------------------|--------------------------------|---------------------------------|
| C <sup>14</sup>   | 5730             | 3                     | (i) _____                      | (ii) _____                      |
| Ra <sup>226</sup> | 1620             | (iii) _____           | (iv) _____                     | 0.4                             |

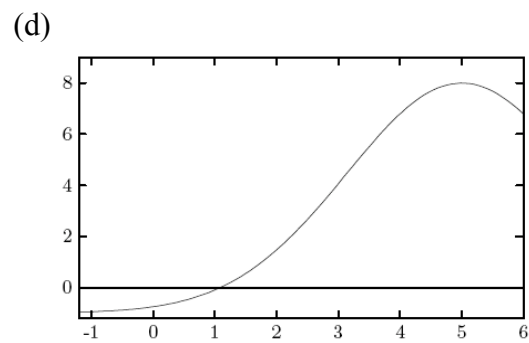
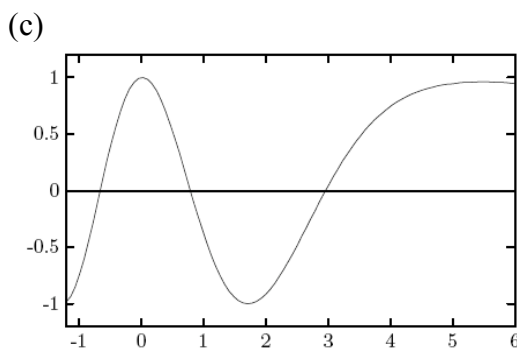
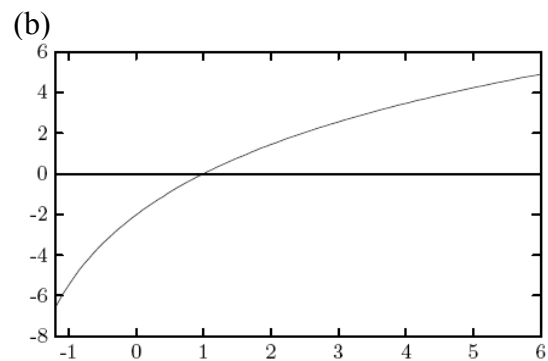
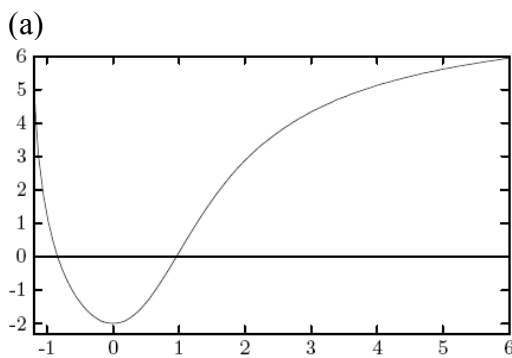
Step-by-step algebra leading to your answer on (i):

Step-by-step algebra leading to your answer on (iii):

5. (15%) Let A be the function defined by  $A(x) = \int_1^x \frac{10t}{2+t^3} dt$  for  $t \geq -\sqrt[3]{2}$ . This sketch might be helpful:



Which of the following is the graph of  $y = A(x)$ ?      **ANS:** \_\_\_\_\_



(End of this exam.)