CALCULUS I (1021)

Midterm Exam

Department of Computer Science and Engineering
National Sun Yat-sen University

November 20, 2013, 13:20 ∼ 15:20

NAME: _____ Student ID No.: _____

Instructor: _____

General instructions:

1. Do not open this exam until you are told to begin.

2. This exam has 7 pages including this cover. There are 6 questions.

3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.

4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.

5. Show an appropriate amount of work for each problem so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.

6. You may use your calculator.

7. Do NOT use pencils but black or blue ball pens for your answers.

8. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to show how you arrived at your solution.

9. Please turn off all cell phones and pagers and remove all headphones.

Problem	1	2	3	4	5	6	**Total**
Points	10	10	20	20	15	25	100
Score							

Name: _____   Student ID No.: _____

No partial credit section. (Prob.1 and 2)

No explanation is necessary. No need to show your work.

1. (10%) Find the limits in the following problems. (2% for each)

(1) $\lim\limits_{x \to 1}(e^{x-1}\sin\frac{\pi x}{2}) = $ _____ .

(2) $\lim\limits_{x \to \infty}\log_{10}(1 + 10^{-x}) = $ _____ .

(3) $\lim\limits_{x \to 9^-}\dfrac{\sqrt{x} - 3}{x - 9} = $ _____ .

(4) $\lim\limits_{x \to 1}\dfrac{x - 1}{x^3 - x^2 + x - 1} = $ _____ .

(5) $\lim\limits_{x \to 0}(2x - 1) = $ _____ .

2. (10%) Find the derivatives in the following problems. (2% for each)

(1) $\dfrac{d}{dx}(xe^x) = $ _____ .

(2) $\dfrac{d}{dx}(\dfrac{\sec x}{x}) = $ _____ .

(3) $\dfrac{d}{ds}(s^3 - 2)^2 = $ _____ .

(4) $\dfrac{d}{dx}2^{3x} = $ _____ .

(5) $\dfrac{d}{dx}(\sqrt{x} + e^x\ln x) = $ _____ .

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3. (20%) Find the limits in the following problems. (4% for each)

(1) $\lim\limits_{\Delta x \to 0} \dfrac{\sin[(\frac{\pi}{6}) + \Delta x] - (\frac{1}{2})}{\Delta x} = $ _____ .

(2) $\lim\limits_{x \to 0} \dfrac{1 - e^{-x}}{e^x - 1} = $ _____ .

(3) $\lim\limits_{x \to 0} \dfrac{x}{x^2 - x} = $ _____ .

(4) $\lim\limits_{x \to -\infty} \dfrac{2x + 1}{\sqrt{x^2 - x}} = $ _____ .

(5) $\lim\limits_{x \to 0} \dfrac{\sqrt{x + 1} - 1}{x} = $ _____ .

4. (20%) Find the derivatives of the following problems. (4% for each)

(1) $y = \ln \sqrt{1 + \sin^2 x}$, $\dfrac{dy}{dx} = $ _____ .

(2) $y = x^{x^x}$, $\dfrac{dy}{dx} = $ _____ .

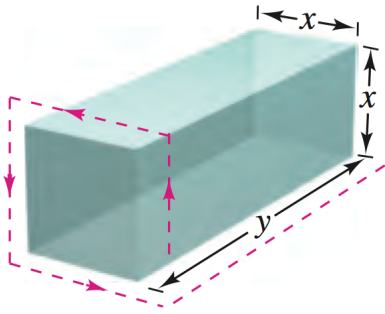(3) $x^2 - 3\ln y + y^2 = 0$, $\dfrac{dy}{dx} = $ _____ .

(4) $y = 2x \arccos x - 2\sqrt{1 - x^2}$, $\dfrac{dy}{dx} = $ _____ .

(5) $y = -\dfrac{1}{a} \ln \dfrac{(a + bx)}{x}$, $a \neq 0$, $\dfrac{dy}{dx} = $ _____ .

5. (15%) A rectangular package to be sent by a postal service can have a maximum combined length and girth (perimeter of a cross section) of 108 inches. Find the dimensions of the package of maximum volume that can be sent. (Assume the cross section is square.)

6. (25%) The point $(0, 4)$ is NOT on the graph of $y = x + \frac{1}{x}$, but it is contained in exactly one tangent line to the graph.

    (a) Use the definition of the derivative as a limit of difference quotients to compute the derivative of $y = x + \frac{1}{x}$ for all points $x > 0$. (10%)

(b) Find the one value of a for which the tangent line to the graph $y = x + \frac{1}{x}$ at $(a, a + 1/a)$ contains (0,4) . (10%)

(c) Write the equation of the corresponding tangent line. (5%)

(End of this exam!)