DISCRETE MATHEMATICS

Final Examination (2014/01/17)

(**<u>You should show how to get your answers in detail or get no credit.</u>**)

1. [10%] (a) Find an Euler circuit in the complete graph K_4. (b) Find all Euler trails in the complete bipartite graph $K_{2,3}$.
2. [10%] Draw all non-isomorphic planar graphs (but not multigraphs) $G = (V, E)$, where $|V| = 4$.
3. [10%] Prove that: Let $G = (V, E)$ be a loop-free graph with $|V| = n > 1$. If $\deg(x)+\deg(y) > n-2$ for all $x, y \in V$, $x \neq y$, then (1) G is connected and (2) G has a Hamilton path.
4. [10%] Find a ring with four elements such that it is not a field, and show why your answer is correct.
5. [10%] Prove that: A finite integral domain is a field.
6. [10%] Find a field with five elements and show why it is a field.
7. [10%] Find $[17]^{-1}$ in $\mathbf{Z}_{180}$?
8. [10%] Find an integer m such that $0 < m < 3 \cdot 4 \cdot 5$ and $\begin{cases} m \equiv 1 \pmod{3} \\ m \equiv 2 \pmod{4} \\ m \equiv 3 \pmod{5} \end{cases}$ by Chinese Remainder Theorem. (<u>Use Chinese Remainder Theorem or get no credit.</u>)
9. [10%] (Fermat's Theorem) If p is a prime, prove that $a^p \equiv a \pmod{p}$ for each $a \in \mathbf{Z}$.
10. [10%] (Euler's Theorem) For each $n \in \mathbf{Z}^+$, $n > 1$, and each $a \in \mathbf{Z}$, prove that if $\mathrm{GCD}(a, n) = 1$, the $a^{\phi(n)} \equiv 1 \pmod{n}$.