線性代數期中考 II 2013.12.04

（最多採記 100 分）

1. (10%) Find $\mathbf{x}_n$ and $\mathbf{x}_r$ where

$$\mathbf{A} = \begin{bmatrix} 1 & -1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

2. (10%) Suppose that $\mathbf{A}$ is 3-by-4, $\mathbf{B}$ is 4-by-5, and $\mathbf{AB} = \mathbf{0}$. Prove that

$$\text{rank}(\mathbf{A}) + \text{rank}(\mathbf{B}) \leq 4.$$

3. (10%) What multiple of $\mathbf{a} = (1, 1, 1)$ is closest to the point $\mathbf{b} = (2, 4, 4)$? Find also the point closest to $\mathbf{a}$ on the line through $\mathbf{b}$.

4. (10%) Find the projection matrix $\mathbf{P}$ onto the space spanned $\mathbf{a}_1 = (1, 0, 1)$ and $\mathbf{a}_2 = (1, 1, -1)$.

5. (10%) For the closest parabola $b = C + Dt + Et^2$ to the points

$$(t, b) = (0, 0), (1, 8), (3, 8), (4, 20)$$

write down the corresponding normal equation.

6. (10%) Find an orthonormal set $\{\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3\}$ for which $\mathbf{q}_1$ and $\mathbf{q}_2$ span the column space of

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ -2 & 4 \end{bmatrix}.$$

7. (10%) Let $\mathbf{A}$ be a 4-by-4 matrix with $|\mathbf{A}| = \frac{1}{2}$. Find $|2\mathbf{A}|$ and $|\mathbf{A}^2|$.

8. (10%) Find the determinant of the matrix

$$\begin{bmatrix} 1+a & b & c & d \\ a & 1+b & c & d \\ a & b & 1+c & d \\ a & b & c & 1+d \end{bmatrix}$$

9. (10%) $\mathbf{A}$ is $m \times n$ and $\mathbf{B}$ is $n \times m$. Show that

$$\begin{vmatrix} \mathbf{0} & \mathbf{A} \\ -\mathbf{B} & \mathbf{I} \end{vmatrix} = |\mathbf{AB}|$$

10. (10%) Suppose permutation matrix $\mathbf{P}$ takes $(1, 2, 3, 4, 5)$ to $(5, 4, 1, 2, 3)$. What does $\mathbf{P}^2$ do to $(1, 2, 3, 4, 5)$? What does $\mathbf{P}^{-1}$ do to $(1, 2, 3, 4, 5)$?

11. (10%) If you know all 16 cofactors of a 4×4 invertible matrix $\mathbf{A}$, how would you find $\mathbf{A}$?