線性代數期中考 I 2013.10.30

1. (10%) For the system of equations

$$\begin{aligned} x + y &= 4, \\ 2x - 2y &= 4. \end{aligned}$$

draw the row picture and the column picture.

2. (10%) Apply elimination (circle the pivots) and back substitution to solve

$$\begin{aligned} 2x - 3y &= 3, \\ 4x - 5y + z &= 7, \\ 2x - y - 3z &= 5, \end{aligned}$$

3. (10%) This 4×4 matrix needs which elimination matrices (to become upper-triangular)?

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}.$$

4. (10%) Solve as two triangular systems, without multiplying $\mathbf{LU}$ to find $\mathbf{A}$:

$$\mathbf{LUx} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

5. (10%) Exchange rows and continue with Gauss-Jordan to find $\mathbf{A}^{-1}$:

$$[\,\mathbf{A}\ \ \mathbf{I}\,] = \begin{bmatrix} 0 & 2 & 1 & 0 \\ 2 & 2 & 0 & 1 \end{bmatrix}.$$

6. (10%) Find the value of c that makes it possible to solve the following system of linear equations, and solve it:

$$\begin{aligned} u + v + 2w &= 2, \\ 2u + 3v - w &= 5, \\ 3u + 4v + w &= c. \end{aligned}$$

7. (10%) Find the dimension and a basis for each of the four fundamental subspaces for

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}.$$

8. (10%) The adjacency matrix $\mathbf{M}$ of a graph has $M_{ij} = 1$ if node i and node j are connected by an edge, and $M_{ij} = 0$ otherwise. Draw of the graph with the following edge-node incident matrix

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix},$$

and find its adjacency matrix $\mathbf{M}$.

9. (10%) The space of all 2×2 matrices has the four basis "vectors"

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

For the linear transformation of *transposing*, find the matrix $\mathbf{A}$ with respect to this basis.

10. (10%) Find the 4×3 matrix $\mathbf{A}$ that represents a *right shift*: (x_1, x_2, x_3) is transformed to $(0, x_1, x_2, x_3)$. Find also the *left shift* matrix $\mathbf{B}$ that transforms (x_1, x_2, x_3, x_4) to (x_2, x_3, x_4).