

Dept. of Computer Science and Engineering, National Sun Yat-sen Univ.

2021 PhD Qualifying Exam

Subject: Probability

1. (20%) A **conservative** team and an **innovative** team are designing a new logo in one week. Suppose the conservative team is successful with probability $1/3$, the innovative team is successful with probability $1/4$, and at least one team is successful with probability $1/2$. Given that exactly one team is successful, what is the probability that it is the innovative team?
2. (20%) Let $G \sim \mathbf{geometric}(1/3)$ be a geometric random variable with parameter $1/3$.
 - Find the expectation of G .
 - Find the expectation of G^2 .
3. (20%) Romeo and Juliet have a date tonight. They will arrive at the meeting place with delays which are independent exponential random variables with expected values of 30 minutes. Find the probability that the difference of their arrival times is within 20 minutes.
4. (20%) 5 persons throw their hats in a box and then each person retrieves one hat at random. Let H be the number of persons that retrieve their own hats.
 - Find the expectation of H .
 - Find the expectation of H^2 .
5. (20%) A professor has 2 umbrellas. Each umbrella is either at home or in his office. He commutes from home to office or from office to home. He commutes without an umbrella if none is available or if it is not raining. Otherwise, he commutes with an umbrella. Let the probability of rain be 0.2. Find the **steady-state probability** that he commutes with an umbrella.