國立中山大學資訊工程學系
108 學年度第 1 學期博士班資格考試

科目：演算法

1. Explain P, NP, NP-hard, NP-complete. (12%)
2. (a) Give the definition of the *longest common subsequence* (LCS) problem. And, then present an example to illustrate your answer. Note that you should give both explanation and example. (5%)

   (b) Design a *dynamic programming* method for calculating the LCS length. (8%)
3. Design an algorithm to solve the *shortest path* problem of a graph. Also give the analysis of the time complexity of the algorithm. (15%)
4. In the self-organizing sequential search heuristics, what are the *transpose heuristics*, *move-to-front heuristics* and *count heuristics*? (15%)
5. An approximate algorithm for solving the *node cover* problem of a graph $G = (V, E)$ is given as follows. Let N denote the solution (node cover). Initially, $F = E$. Arbitrarily select an edge $(u,v) \in F$, then add nodes u and v into N. Next, remove all edges incident to u or v from F. Repeat the above procedure until F becomes empty. Suppose that C is the size of the optimal solution (node cover). Show that $|N| \leq 2C$. (15%)
6. Prove that the *sum of subsets* decision problem polynomially reduces to the *partition* decision problem. (15%)
7. It is interesting whether there exist three integers a, b, c for the equation $a^3+b^3+c^3=y$, where y is a given integer. This problem is called the *sum of three cubes* problem. The answer is YES for y=3 or y=2. For example,

   $1^3+1^3+1^3=4^3+4^3+(-5)^3=3$;

   $1^3+1^3+0^3=1214928^3+3480205^3+(-3528875)^3=2$.

   In September 2019, the number y=42 was solved. This success completes the solution of each number between 1 and 100. It was also known that there is no solution for some values of y (described in subproblem b) before y=42 was solved. Prove the following:

   (a) The cube of any integer modulo 9 is 1, -1, or 0. (10%)

   (b) If y modulo 9 is 4 or 5, then there is no solution for y in the sum of three cubes problem. (5%)