Final exam 2018.06.20

1. (20%) To win a badminton game, a player needs 21 points. When the score is tied at 20, the winner needs two more points than his opponent, or get 30 points. In a game of a final match, suppose each point is won by either side **equally likely**, and the score is 19:18.

   - What is the probability that the player who is behind wins the game?
   - What is the probability that the final score is 21:19? (Note either player can win with this score.)

2. (20%) In World Cup, a death group has 4 teams (including Team X) of **equal strength**. A team has a match against every other team in the group. The winner of a match gets 3 points, and the loser gets 0. With a draw, both teams get 1 point. A match is a draw with probability 1/3. After 6 matches, the two teams with the most accumulated points advance, with ties broken equally likely.

   - What is the PMF of Y, which is the sum of accumulated points of the entire group?
   - Team X has 3 draws. What is the probability that it advances?

3. (20%) A surface is ruled horizontally and vertically by a space of 5cm. A needle of length 3cm is dropped on the surface at random.

   - What is the probability that the needle intersects a horizontal line?
   - What is the probability that the needle intersects two lines?
   - What is the probability that the needle intersects just one line?
   - What is the probability that the needle intersects no lines?

4. (20%) $X \sim$ **uniform**$(0, 1)$ and $Y = e^X$.

   - What is $E[Y]$?
   - What is PDF f_Y?

5. (20%) You call the IRS hotline. You are the 56th person in line, excluding the person being served. Assume that the inter-departure times of the callers are **i.i.d.** exponential random variables with parameter $\lambda = 2$.

   - How long will you have to wait on average until your service starts?
   - Approximate the probability that you wait for more than 30 minutes in terms of $\Phi(y)$, the CDF of a standard normal. (central limit theorem)