Midterm 2016.04.20

1. (10%) Draw the CDF of

   (a) a continuous random variable X with PDF

   $$f_X(x) = \begin{cases} \frac{1}{2\sqrt{x}}, & 0 < x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

   (b) a discrete random variable Y with PMF

   $$p_Y(k) = \left(\frac{k}{5}\right)^2 - \left(\frac{k-1}{5}\right)^2, \quad k = 1, \ldots, 5$$

2. (10%) A train arrives at a station every quarter hour starting at 6:00 a.m. You walk into the station every morning between 7:10 a.m. and 7:40 a.m., and your arrival time is a uniform random variable over this interval. What is the expectation of the time you have to wait for the first train to arrive?

3. (10%) A binary signal S is transmitted, and we are given that $\mathbf{P}(S = 1) = 1/2$ and $\mathbf{P}(S = -1) = 1/2$. The received signal is $Y = S + N$, where N is a normal noise with zero mean and unit variance. What is the probability that $S = 1$, as a function of the observed value y of Y? Plot this function.

4. (10%) Every school day, Adam flips a fair coin to decide how to get to school. If a head shows up, he walks, taking 10-20 minutes. If a tail shows up and is followed by a head, he rides on his motorcycle, 2-5 minutes. If 2 tails show up in a row, he bikes, 5-10 minutes. All times are uniform. What is the expectation of the time T for Adam to get to school?

5. (10%) Consider a game where a person is given 2 questions and can select which one to answer first. Question A is answered correctly with probability 0.6, and the prize money is 100. Question B is answered correctly with probability 0.8, and the prize money is 200. If the first question attempted is answered incorrectly, the quiz terminates. If it is answered correctly, the person is allowed to attempt the second question to earn more money. What are the expected value of the total prize money received if

   (a) Question A is answered first?

   (b) Question B is answered first?

6. (10%) Klay Thompson attempts a 3-pointer and makes it with probability 0.5, independent of other attempts. In each game, he attempts 3-5 times with equal probability 1/3. What is the probability that he does not make any 3-pointers in a game?

7. (10%) Consider a continuous random variable X uniformly distributed over the interval $[1, 3]$. What is the expectation and variance of X?

8. (10%) The number of e-mails Bill gets in one hour is a Poisson random variable with parameter $\lambda = 0.5$. He checks e-mails every 3 hours. What is the probability of finding 1 new message?

9. (20%) A surface contains repeated squares with side d. Suppose that a needle of length l is dropped on the surface at random. Assume that $l < d$. What is the probability that the needle intersects 2 sides of a square?