Final exam 2016.06.15

1. (10%) 阿明有 125 張發票。對獎之前，阿明找阿光對賭

   - 如果槓龜，阿光給阿明 50 元
   - 否則，阿明給阿光 100 元

   中獎號碼有四組，中末三碼獎金 200 元，別的獎機率太小忽略不計。分析一下阿光的期望值是正的還是負的？

2. (10%) 高捷少女扭蛋機扭一次10元，裡面有六個公仔。假設每次每種公仔被扭到的機率一樣，請問集完全部公仔所需成本期望值爲何？

3. (10%) In a go tournament, your probability of winning the first game is 0.5 against half the players, 0.6 against one third of the players, and 0.3 against the remaining players. You play the first game against a randomly chosen opponent. What is the probability of winning the first game?

4. (10%) A student takes a test until he passes the test, each time with a probability 0.5 of passing, independent of the previous attempts, up to a maximum number of 3 times. What is the conditional PMF of the number of attempts K, given that the student passes the test?

5. (10%) Find the 3rd moment of a geometric random variable with mean 2.

6. (10%) You and your partner go to a badminton court. There are 2 groups waiting. Assume that the playing time of a group is an exponential random variable with mean 20 minutes. What is the PDF of your waiting time?

7. (10%) Klay Thompson attempts a 3-pointer and makes it with probability 0.6, independent of other attempts. In each game, he attempts 2-4 times with equal probability 1/3. What is the probability that he does not make any 3-pointers in a game?

8. (10%) A machine can be either working or broken down on a given day.

   - If it is working, it will break down the next day with probability 0.3, or it will continue to be working with probability 0.7.
   - If it breaks down, it will be working the next day with probability 0.6, or it will continue to be broken down with probability 0.4.
   - However, if the machine remains broken down for 3 consecutive days, it will always be working the next day.

   Construct a Markov chain.

9. (10%) An absent-minded professor has 1 umbrellas that he uses when commuting from home to office and back. If it rains and an umbrella is available in his location, he takes it. If it is not raining, he always forgets to take an umbrella. Suppose that it rains with probability 0.3 each time he commutes. What is the steady-state probability that he gets wet during a commute?

10. (10%) 2 light bulbs have independent exponentially distributed lifetimes with the means of 1000 hours and 500 hours respectively. They are switched on at time 0. What is the expected time to the first burn-out time?

11. (10%) Alice walks along a straight line and, at each time period, takes a step to the right (position +1) with probability 0.3, and a step to the left with probability 0.7. She starts in one of the positions $1, 2, 3$, but if she reaches the position 0 (or position 4), her step is instantly reflected back to 1 (or 3, respectively). We introduce a Markov chain model whose states are the positions $1, 2, 3$. What are the steady-state probabilities?

12. (10%) The pair of random variables (X, Y) takes the values of

$$(2, 0), (0, 2), (-1, 0), (0, -1)$$

each with probability 1/4. What is the covariance of X and Y?