國立中山大學資訊工程學系

108學年度第1學期博士班資格考試

科目：機率學

1. (20%) **Romeo** and **Juliet** have a date, and each will show up with a delay between 0 and 1 hour, with all pairs of delays being equally likely. The first to arrive will wait for 20 minutes and will leave if the other has not yet arrived. What is the probability that they miss each other?

2. (20%) A **conservative** team and an **innovative** team are asked to design a new product within a month. Suppose the conservative team is successful with probability 1/3, the innovative team is successful with probability 1/2, and at least one team is successful with probability 2/3. Given that exactly one team is successful, what is the probability that it is the innovative team?

3. (10%) Suppose $X \sim$ **uniform**$(0, 1)$ and $Y = X^3$. What is the **probability density function** of Y?

4. (20%) A computer executes 2 types of jobs, **priority** and **non-priority**. A slot is **busy** if the computer executes a priority job, **idle** otherwise. We call a string of idle slots, flanked by busy slots, an **idle period**. We call a string of busy slots, flanked by idle slots, a **busy period**. A priority job occurs with probability 0.2 at the beginning of each slot, independent of other slots, and requires one full slot to execute. A non-priority job is always available and is executed at a given slot if no priority job is available.

   Let T be the time of the first idle slot, B be the length of the first busy period, I be the length of the first idle period, and Z be the number of slots after the first slot of the first busy period, including the first subsequent idle slot. What are their **expected values** and **variances**?

5. (10%) Three light bulbs have lifetimes which are independent **exponential** random variables, each with a mean of 10 days. They are turned on at the same time. What is the expected time until the last bulb burns out?

6. (20%) An absent-minded professor has 2 umbrellas that he uses when commuting from home to office and back. Suppose that it rains with probability 0.1 each time he commutes. What is the **steady-state probability** that he gets wet during a commute?