線性代數期末考 2015.01.14

每題 10 分，最多採計 100 分。作答要寫過程，不要只寫答案。

1. What 3 by 3 matrix adds row 1 to row 3 and then adds row 3 to row 1?

2. The cosine space $\mathbf{F}_3$ contains all combinations

$$y(x) = A\cos x + B\cos 2x + C\cos 3x.$$

   Find a basis for the subspace that has $y(0) = 0$.

3. If you know all 16 cofactors of a 4 by 4 invertible matrix $\mathbf{A}$, how would you find $\mathbf{A}$?

4. Find a real 2×2 matrix $\mathbf{A} \neq \mathbf{I}$ with

$$\mathbf{A}^3 = \mathbf{I}.$$

5. Find the determinant of

$$\begin{vmatrix} 1+a & b & c & d \\ a & 1+b & c & d \\ a & b & 1+c & d \\ a & b & c & 1+d \end{vmatrix}.$$

6. Let F_n be the determinant of the $1, 1, -1$ tridiagonal matrix of size $n \times n$:

$$F_n = \begin{vmatrix} 1 & -1 & & & \\ 1 & 1 & -1 & & \\ & 1 & 1 & -1 & \\ & & \cdot & \cdot & \cdot \\ & & & 1 & 1 \end{vmatrix}.$$

   Show that

$$F_n = F_{n-1} + F_{n-2}.$$

7. Show that

$$\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}^k = \begin{bmatrix} 3^k & 3^k - 2^k \\ 0 & 2^k \end{bmatrix}$$

   through diagonalization or mathematical induction.

8. Find the eigenvalues and the eigenvectors for the following matrix

$$\mathbf{A} = \begin{bmatrix} 0.2 & 0.4 & 0.3 \\ 0.4 & 0.2 & 0.3 \\ 0.4 & 0.4 & 0.4 \end{bmatrix}.$$

9. Compute $\mathbf{A}^H\mathbf{A}$ and $\mathbf{A}\mathbf{A}^H$:

$$\mathbf{A} = \begin{bmatrix} i & 1 & i \\ 1 & i & i \end{bmatrix}.$$

10. Under what conditions on a, b, c is

$$ax^2 + 2bxy + cy^2 > x^2 + y^2$$

for all x, y?

11. The Cholesky factorization for a positive definite matrix is defined by

$$\mathbf{A} = \mathbf{C}\mathbf{C}^T,$$

where $\mathbf{C}$ is lower-triangular. With the help of LDU decomposition, find $\mathbf{C}$ for

$$\mathbf{A} = \begin{bmatrix} 4 & 8 \\ 8 & 25 \end{bmatrix}.$$

12. Find the SVD of the following matrices

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$