CALCULUS I (1031)

Final Exam

Department of Computer Science and Engineering
National Sun Yat-sen University

January 14, 2015, 13:20 ∼ 15:30

NAME: Solution Student ID No.: _____

Instructor: _____

General instructions:

1. Do not open this exam until you are told to begin.
2. This exam has 7 pages including this cover. There are 5 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each problem so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. No calculator is allowed.
7. Do NOT use pencils but black or blue ball pens for your answers.
8. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to show how you arrived at your solution.
9. Please turn off all cell phones and pagers and remove all headphones.

Problem	1	2	3	4	5	**Total**
Points	40	20	15	15	10	100
Score						

Name: _____ Student ID No.: _____

Some useful trigonometric identities and formulas

1. $\sin^2 x = \dfrac{1 - \cos 2x}{2}$
2. $\cos^2 x = \dfrac{1 + \cos 2x}{2}$
3. $\sin mx \sin nx = \dfrac{1}{2}(\cos[(m-n)x] - \cos[(m+n)x])$
4. $\cos mx \cos nx = \dfrac{1}{2}(\cos[(m-n)x] + \cos[(m+n)x])$
5. $\sin mx \cos nx = \dfrac{1}{2}(\sin[(m-n)x] + \sin[(m+n)x])$
6. $\displaystyle\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$
7. $\displaystyle\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$

Name: _____ Student ID No.: _____

1. (40%) Short answer. (4% for each, NO PARTIAL CREDIT!) Mark your answer as "diverge" if the improper integral is either ∞ or $-\infty$.

(1) Evaluate the integral $\int_{-2}^{1} x(x^2-6)\,dx =$ $\frac{21}{4}$

(2) Evaluate the integral $\int_{1}^{2} (\frac{3}{x^2}-1)\,dx =$ $\frac{1}{2}$

(3) Find the integral $\int e^x(e^x+1)^2\,dx =$ $\frac{(e^x+1)^3}{3}+C = \frac{e^{3x}}{3}+e^{2x}+e^x+C$

(4) Evaluate the integral $\int_{0}^{5} (5-|x-5|)\,dx =$ $\frac{25}{2}$

(5) Find the integral $\int \frac{\sin x}{\cos^3 x}\,dx =$ $\frac{1}{2}\sec^2 x + C = \frac{1}{2}\tan^2 x + C$

(6) Find the integral $\int 2x\sqrt{2x-3}\,dx =$ $\frac{2}{5}(2x-3)^{\frac{3}{2}}(x+1)+C$

(7) Evaluate the integral $\int_{0}^{1} \frac{x^2-x}{x^2+x+1}\,dx =$ $1-\ln 3$

(8) Evaluate the integral $\int_{1}^{\infty} \frac{\ln x}{x}\,dx =$ *diverge*

(9) Evaluate the integral $\int_{2}^{4} \frac{1}{\sqrt{x^2-4}}\,dx =$ $\ln(2+\sqrt{3})$

(10) $F(x) = \int_{0}^{x^3} \sin(t^2)\,dt$, $F'(x) =$ $3x^2\sin(x^6)$

Name: _____ Student ID No.: _____

2. (20%) Evaluate the integrals of the following problems. (5% for each)

(1) $\int x\sin(3x^2)\,dx =$ $\underline{-\frac{1}{6}\cos(3x^2)+C}$

$$\int x\sin(3x^2)\,dx = \frac{1}{6}\int \sin(3x^2)\,d(3x^2) = -\frac{1}{6}\cos(3x^2)+C$$

(2) $\int x^2\sin x\,dx =$ $\underline{-x^2\cos x + 2x\sin x + 2\cos x + C}$.

$$\int x^2\sin x\,dx = \int(-x^2)\,d\cos x = (-x^2)\cos x - \int \cos x\,d(-x^2)$$
$$= (-x^2)\cos x + 2\int \cos x\,dx^2 = (-x^2)\cos x + 2\int x\cos x\,dx$$
$$= (-x^2)\cos x + 2\int x\,d\sin x = (-x^2)\cos x + 2(x\sin x - \int \sin x\,dx)$$
$$= -x^2\cos x + 2x\sin x + 2\cos x + C$$

(3) $\int_{-\infty}^{\infty}\frac{e^x}{1+e^{2x}}\,dx =$ $\underline{\frac{\pi}{2}}$

$$\int_{-\infty}^{\infty}\frac{e^x}{1+e^{2x}}\,dx = \lim_{b\to-\infty}[\arctan e^x]_b^0 + \lim_{b\to\infty}[\arctan e^x]_0^b$$
$$= \lim_{b\to-\infty}(\frac{\pi}{4} - \arctan e^b) + \lim_{b\to\infty}(\arctan e^b - \frac{\pi}{4}) = \frac{\pi}{4} - 0 + \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{2}$$

(4) $\int_1^{\infty}(1-x)e^{-x}\,dx =$ $\underline{-\frac{1}{e}}$

$$\int_1^{\infty}(1-x)e^{-x}\,dx = \lim_{b\to\infty}\int_1^b (x-1)\,de^{-x}$$
$$= \lim_{b\to\infty}\{(x-1)e^{-x}]_1^b - \int_1^b e^{-x}\,d(x-1)\} = \lim_{b\to\infty}\left[(x-1)e^{-x} + e^{-x}\right]_1^b$$
$$= \lim_{b\to\infty}(be^{-b} - e^{-1}) = \lim_{b\to\infty}\frac{b}{e^b} - \frac{1}{e} = \lim_{b\to\infty}\frac{1}{e^b} - \frac{1}{e} = -\frac{1}{e}$$

Name: _____ Student ID No.: _____

3. (15%) Fine the arc length of the graph: $y = \frac{x^3}{6} + \frac{1}{2x}$, on the interval $\frac{1}{2} \le x \le 2$.

$$\frac{dy}{dx} = \frac{3x^2}{6} - \frac{1}{2x^2} = \frac{1}{2}(x^2 - \frac{1}{x^2})$$

$$S = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2}\, dx = \int_{1/2}^2 \sqrt{1 + \left[\frac{1}{2}(x^2 - \frac{1}{x^2})\right]^2}\, dx$$

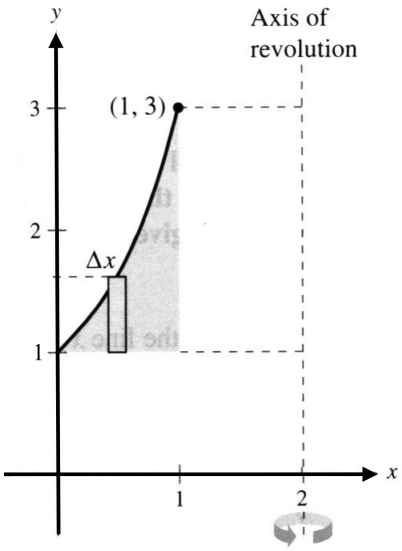
$$= \int_{1/2}^2 \sqrt{\frac{1}{4}(x^4 + 2 + \frac{1}{x^4})}\, dx = \int_{1/2}^2 \frac{1}{2}(x^2 + \frac{1}{x^2})\, dx$$

$$= \frac{1}{2}\left[\frac{x^3}{3} - \frac{1}{x}\right]_{1/2}^2$$

$$= \underline{\frac{33}{16}}$$

Name: _____ Student ID No.: _____

4. (15%) Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^3 + x + 1$, $y = 1$, and $x = 1$ about the line $x = 2$. as shown in the figure.



$$V = 2\pi \int_a^b p(x)h(x)\ dx = 2\pi \int_0^1 (2-x)(x^3+x+1-1)\ dx$$

$$= 2\pi \left[-\frac{x^5}{5} + \frac{x^4}{2} - \frac{x^3}{3} + x^2 \right]_0^1$$

$$= \underline{\frac{29}{15}\pi}$$

Name: _____ Student ID No.: _____

5. (10%) Find: $\int \sin(5x)\cos(4x)\ dx$.

$$\int \sin(5x)\cos(4x)\ dx = \frac{1}{2}\int (\sin x + \sin 9x)\ dx$$

$$= \frac{1}{2}\left(-\cos x - \frac{\cos 9x}{9}\right) + C$$

$$= \underline{\quad -\frac{\cos x}{2} - \frac{\cos(9x)}{18} + C \quad}$$

(End of this exam!)
Have a nice vacation!