DISCRETE MATHEMATICS

Final Examination (2015/01/16)

(**You should show how to get your answers in detail or get no credit.**)

1. [10%] (a) Find an Euler circuit in the complete graph K_5. (b) Find all Euler trails in the complete bipartite graph $K_{2,3}$.
2. [10%] Draw all non-isomorphic planar graphs (but not multigraphs) $G = (V, E)$, where $|V| = 4$.
3. [10%] Find a ring with 12 elements such that it is not a field, and show why your answer is correct.
4. [10%] What is ring homomorphism? What is ring isomorphism?
5. [10%] Find a ring with proper divisors of zero.
6. [10%] Find a field with 13 elements and show why it is a field.
7. [10%] Find $[13]^{-1}$ in $\mathbf{Z}_{150}$?
8. [15%] Find an integer m such that $0 < m < 3 \cdot 5 \cdot 7$ and $\begin{cases} m \equiv 1 \pmod 3 \\ m \equiv 2 \pmod 5 \\ m \equiv 3 \pmod 7 \end{cases}$ by Chinese Remainder Theorem. (Use Chinese Remainder Theorem or get no credit.)
9. [15%] Prove that: Let $G = (V, E)$ be a loop-free graph with $|V| = n > 1$. If $\deg(x)+\deg(y) > n-2$ for all $x, y \in V, x \neq y$, then (1) G is connected and (2) G has a Hamilton path.