[Midterm Test, 2014/11/28]

Discrete Mathematics

1. [10%] Let $\Sigma = \{a, b, c, x, y\}$. (a) What is $|\Sigma^6|$? (b) How many strings in Σ^* have length at most 6?
2. [9%] Let $\Sigma = \{a, b, c\}$, and consider the string $w = aabcc$. Please list (a) all prefixes, (b) all proper suffixes, and (c) all substrings of w.
3. [10%] (a) Construct a state diagram for a finite state machine with $I = O = \{0, 1\}$ that recognizes all strings in the language $\{11\}\{0, 1\}^*\{00\}$, where I is the input alphabet and O is the output alphabet of the machine. (b) Construct a state diagram for a finite state machine that recognizes all strings in $\{11\}\{01\}^*\{00\}$.
4. [15%] Let A be a set with $|A| = n$. (a) How many binary relations on A are reflexive? (b) How many binary relations on A are symmetric? (c) How many binary relations on A are reflexive and symmetric? (d) How many binary relations on A are reflexive but not symmetric? (e) How many binary relations on A are antisymmetric and reflexive?
5. [8%] Let R be the "(exactly) divides" relation defined on $A = \{1, 3, 5, 6, 7, 11, 12, 35, 385\}$. Please draw the Hasse diagram for the poset (A, R).
6. [10%] (a) If $A = \{1, 2, 3, 4, 5, 6, 7\}$ and R is the equivalence relation on A that induces the partition $A = \{6, 7\} \cup \{1, 2, 4\} \cup \{3, 5\}$, what is R? (b) If R is the equivalence relation on A that induces the partition $A = A_1 \cup A_2 \cup \cdots \cup A_n$ where $n>2$ is a positive integer and $|A_i| = i$ for each i in $\{1, 2, \ldots, n\}$, then $|R| = ?$
7. [10%] Let $A = \{1, 2, 3, 4, 5\} \times \{1, 2, 3, 4, 5\}$, and define R on A by $(x_1, y_1)\ R\ (x_2, y_2)$ if $((x_1 + y_1) \bmod 5) = ((x_2 + y_2) \bmod 5)$ (a) Determine the equivalence class $[(1, 1)]$ and (b) determine the partition of A induced by R.
8. [8%] (a) If $A = \{1, 2, 3, 4, 5\}$, how many relations on A are equivalence relations? (b) How many of the equivalence relations in part (a) satisfy $1 \in [2]$?
9. [20%] Minimize the two finite state machines defined in Table 7.4 and Figure 7.26, respectively.

Table 7.4

	v		ω	
	0	1	0	1
s_1	s_4	s_1	0	1
s_2	s_3	s_3	1	0
s_3	s_1	s_4	1	0
s_4	s_1	s_3	0	1
s_5	s_3	s_3	1	0

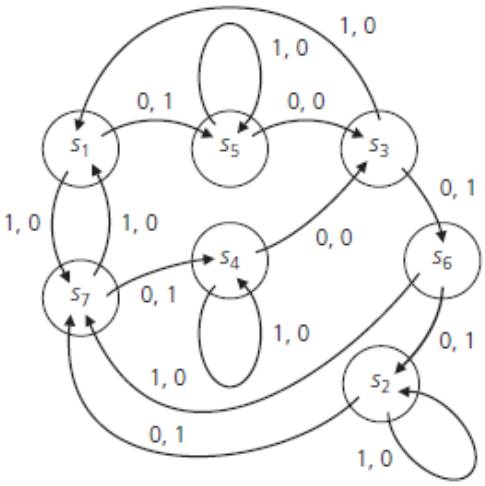

Figure 7.26

[You should show how to get the answers in detail or obtain no credit.]