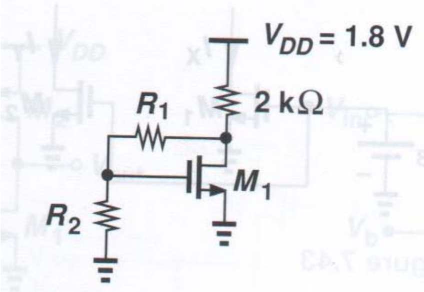
Digital Electronics Spring 2015 Final Exam 06/26/2015

Name: ID#

$\mu_n C_{ox}$ =200 μA/V$^2$ , $\mu_p C_{ox}$ =100 μA/V$^2$ , NMOS $V_{TH}$=0.4 V, PMOS $V_{TH}$=–0.4 V,

$I_D$=(1/2) $\mu_n C_{ox}$ (W/L)( $V_{GS}$-$V_{TH}$)$^2$ , $g_m$=[2$\mu_n C_{ox}$ (W/L) $I_D$]$^{1/2}$ , $r_o$=[1/( $\lambda I_D$ )]

1. (10%) $I_D$=0.5 mA, W/L = 50/0.18. Find $R_1$ and $R_2$ such that current of resistors is 0.05mA..




Ans:

$$I_{DS} = \frac{1}{2}(\mu_n C_{ox})\left(\frac{W}{L}\right)(V_{GS} - V_{TN})^2$$

$$0.5 \times 10^{-3} = (100 \times 10^{-6})\left(\frac{50}{0.18}\right)(V_{GS} - V_{TH})^2$$

$$\therefore V_{GS} = 0.534\,V$$

$$\therefore R_2 = \frac{0.534}{0.05 \times 10^{-3}}$$

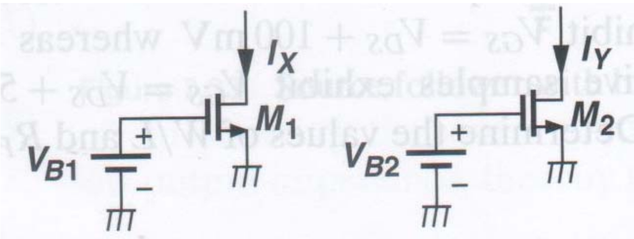
$$R_2 = 10.68\,k\Omega$$

$$\because V_{D1} = 1.8 - (1.1 \times I_{DS} \times 2\,k\Omega) = 0.1\,I_{DS}(R_1 + R_2)$$

$$\therefore 14\,k\Omega = R_1 + 10.68\,k\Omega$$

$$\therefore R_1 = 3320\,\Omega$$

2. (10%) $I_X$= $I_Y$= 0.6 mA. If $V_{B1}$ = 1.1 V, $V_{B2}$ = 1.0 V, λ = 0.1 V$^{-1}$ and $L_1$ = $L_2$= 0.25um, calculate $W_1$ and $W_2$. Calculate output resistance of these current sources.




Ans:

$$I_x = \frac{\beta_n}{2}(V_{GS} - V_{tn})^2$$

$$0.6\,\text{mA} = \frac{\mu_n C_0 \times W_1}{2 \times L_1}(V_{in} - V_{tn})^2 = \frac{200 \times 10^{-6}}{2} \times \frac{W_1}{0.25 \times 10^{-6}} \times (1.1 - 0.4)^2 = 196 W_1$$

$$\Rightarrow W_1 = 3\ \mu\text{m}.$$

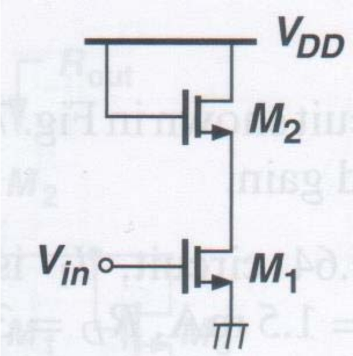
$$I_y = \frac{\beta_n}{2}(V_{GS} - V_t)^2 = \frac{200 \times 10^{-6}}{2} \times \frac{W_2}{0.25 \times 10^{-6}} \times (1 - 0.4)^2 = 144 W_2$$

$$\Rightarrow W_2 = \frac{0.6\,\text{mA}}{144} = 4\ \mu\text{m}.$$

es

$$R_{out1} = R_{out2} = \frac{1}{\lambda I_D} = \frac{1}{0.1 \times 0.6 \times 10^{-3}} = 16.67\ \text{k}\Omega.$$

3. (10%) $I_D = 1$ mA, $(W/L)_2 = 5/1$, $(W/L)_1 = 10/1$, $\lambda_1 = 0.1\ V^{-1}$, $\lambda_2 = 0.1\ V^{-1}$, calculate $R_{out}$




Ans:

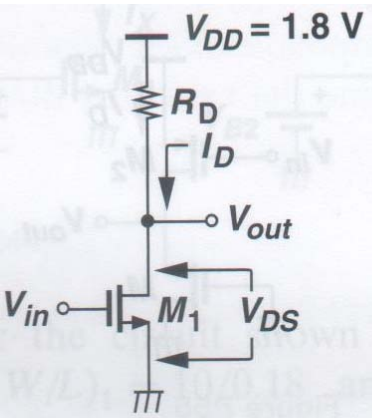
$$r_{O2} = \frac{1}{\lambda I_D} = \frac{1}{0.1 \times 10^{-3}} = 10\,\text{k}\Omega.$$

$$r_{O1} = \frac{1}{\lambda I_D} = 10\,\text{k}\Omega.$$

$$g_{m_2} = \sqrt{2 \times 200 \times 10^{-6} \times \frac{5}{1} \times 1 \times 10^{-3}} = 0.00141\text{S}.$$

$$R_{out} = \frac{1}{g_{m_2}} \| r_{O2} \| r_{O1} = 709 \| 10\,\text{k}\Omega \| 10\,\text{k}\Omega \approx 709\,\Omega.$$

4. (10%) determine the gate voltage at which $M_2$ operates at the edge of saturation.




Ans:

**Solution** We know at the edge of saturation of $M_1$

$$(V_{GS} - V_t) = V_{DS}.$$

Here,

$$(V_{in} - 0) = V_{GS}.$$

So, $(V_{in} - V_t) = V_{DS}$. Therefore,

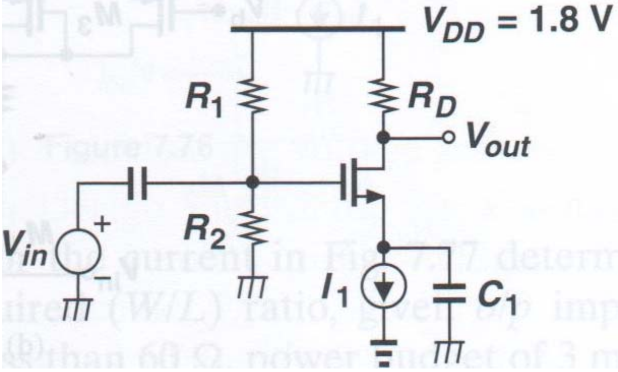
$$V_{DS} = V_{DD} - I_D R_D$$
$$= 1.8 - I_D R_D.$$

As

$$(V_{in} - V_t) = 1.8 - I_D R_D$$

$$\Rightarrow \text{Gate voltage } \boxed{V_{in} = 1.8 - I_D R_D + V_t}$$

5. (10%) $I_1$ = 1.5 mA, $R_D$ = 350Ω . λ= 0, C is large, compute (W/L) to obtain gain of 7. $A_v = g_m R_D$




Ans:

**Solution** $I_1 = 1.5$ mA, $R_D = 350\ \Omega$, $\lambda = 0$, $I_1$ ideal source, $(W/L) = ?$ for $A_V = 7$.

$$I_1 = \frac{1}{2}\mu_n C_0 \times \frac{W}{L}\cdot(V_{G_S} - V_t)^2.$$

$$g_m = \sqrt{2\mu_n C_0 \times \frac{W}{L} I_D} = \sqrt{2\times 200\times 10^{-6}\times \frac{W}{L}\times 1.5\times 10^{-3}} \quad (1)$$

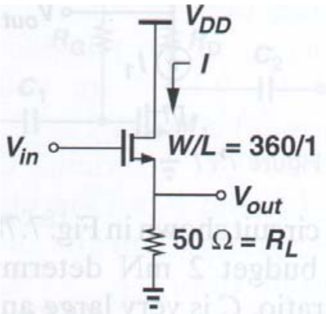
$$A_V = g_m \cdot R_D \Rightarrow 7 = g_m(350) \quad (2)$$

$g_m = 7/350$. Putting $g_m$ value in Eq. (1), we get

$$\frac{7}{350} = \sqrt{2\times 200\times 10^{-6}\times \frac{W}{L}\times 10^{-3}} \Rightarrow \frac{W}{L} = 667.$$

6. (10%) Transistor with W/L = 360, $R_L$ = 50Ω, power is 20 mW, find voltage gain ( $V_{DD}$ = 2 V).

$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L} I_D}$$



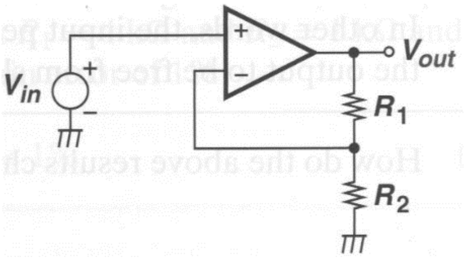

Ans:

**Solution** $p = 20\,\text{mW}; V_{DD} = 2V \Rightarrow \max I = \dfrac{20\times 10^{-3}}{2V} = 10\,\text{mA}$

**We know** $$g_m = \sqrt{2\mu_n C_0 \times \frac{W}{L} I_D} = \sqrt{2\times 200\times 10^{-6} 360\times 10\times 10^{-3}} = 0.0379\text{S}$$

**We have** $$A_v = \frac{R_L}{\frac{1}{g_m} + R_L} = \frac{50}{26.38 + 50} = 0.65\ \text{v/v}.$$

7. (10%) For noninverting amplifier, Find the expression for $V_{out}$ ..




Ans:

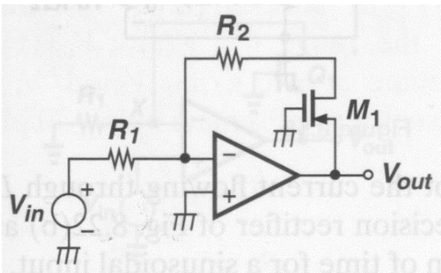
$$\left[\left(V_{in}-V_{out}\frac{R_2}{(R_1+R_2)}\right)A_0-V_{out}\right]\frac{1}{R_{out}}=\frac{V_{out}}{R_1+R_2}$$

Since, $R_{out} << (R_1+R_2)$, so,

$$\frac{V_{out}}{V_{in}}=\frac{A_0}{\left[1+A_0\cdot\frac{R_2}{(R_1+R_2)}\right]}$$

$$\Rightarrow V_{out}=\frac{A_0V_{in}}{\left[1+A_0\frac{R_2}{(R_1+R_2)}\right]}$$

8. (10%) Derive the expression for output voltage in terms of input voltage.




ANS:

$$i_1=\frac{V_i-V_A}{R_1}=\frac{V_i}{R_1} \qquad (1)$$

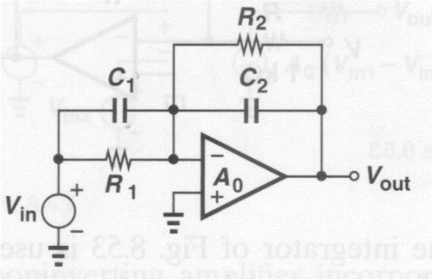
$$i_2=\frac{V_A-V_{out}}{R_2}=\frac{-V_{out}}{R_2} \qquad (2)$$

$$i_1 \,\square\, i_2 \Rightarrow \frac{V_i}{R_1}=\frac{-V_{out}}{R_2}$$

$$\frac{V_{out}}{V_i}=\frac{-R_2}{R_1}.$$

Vout= (-R2/R1)Vi

9. (10%) calculate the transfer function of the circuit shown below if $A_0 = \infty$ . What choice of component values reduces | $V_{out}$ / $V_{in}$ |to unity at all frequencies. $X_{C1}$=1/(sC1), $X_{C2}$=1/(sC2)




Ans:

$\because A_0 = \infty,$

$V_+ = V_- = 0$

By KCL,

$$\frac{V_{in}}{R_1 \,//\, \frac{1}{sC_1}} = -\frac{V_{out}}{R_2 \,//\, \frac{1}{sC_2}}$$

$$\therefore \frac{V_{out}}{V_{in}} = -\frac{R_2 \,//\, \frac{1}{sC_2}}{R_1 \,//\, \frac{1}{sC_1}}$$

$$= -\frac{R_2}{R_1} \times \frac{1 + R_1 C_1 s}{1 + R_2 C_2 s}$$

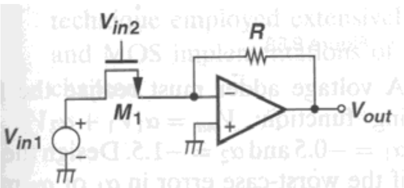
For $\left|\frac{V_{out}}{V_{in}}\right| = 1$,

$$R_2 \,//\, \frac{1}{sC_2} = R_1 \,//\, \frac{1}{sC_1}$$

That is, choose the components such that the impedance of $R_2 // \frac{1}{sC_2}$ is equal to $R_1 // \frac{1}{sC_1}$ at the specific frequency. //

10.(10%) analyze the function of a circuit and verify its function mathematically..

$$R_{on} = \frac{1}{\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{TH})}$$




ANS:

**Solution** In negative feedback circuits another resistance is replaced by $M_1$. We know that

$$\frac{V_0}{V_{in}} = \frac{-R_2}{R_1}$$

is that case. $R_1$ is now equivalent to resistance of $M_1$. Here we must know minimum $V_{in2}$ required, which will provide equivalent resistance of $R_1$ for required gain. Based on this we can get acceptable range of $V_{in1}$ values, ensuring the required gains.

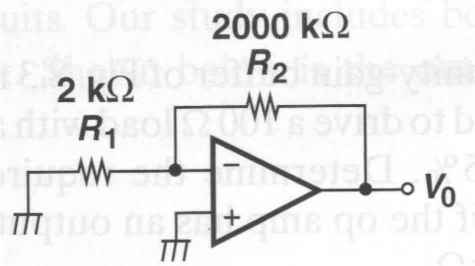
For Fig. 8.60, we can write

$$\frac{V_{out}}{V_{in1}} = \frac{-R_2}{R_1} = \frac{-R_2}{1/kn'\frac{W}{L}V_{in2}} = -kn'\frac{W}{L}V_{in2}R_2$$

$\Rightarrow V_{out} = (V_{in} \cdot V_{in2})\left(K_n^1 \frac{W}{L} R_2\right)$ ~~$\Rightarrow V_{in2}$ should be at least $(V_{GS} - V_{TH})$~~ Vin2 at least (VGS-VTH)

$\Rightarrow$ Circuit works as a multiplier; $\Rightarrow$ o/p voltage proportional to product of $V_{in1}$ and $V_{in2}$.

11. (10%) calculate output voltage $V_O$ for input offset voltage of 2 mV.




(a)

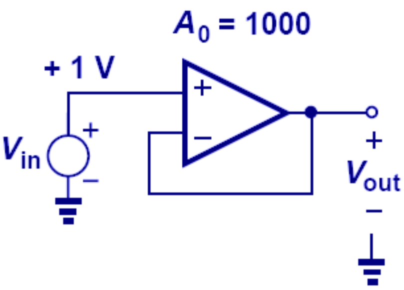
Ans:

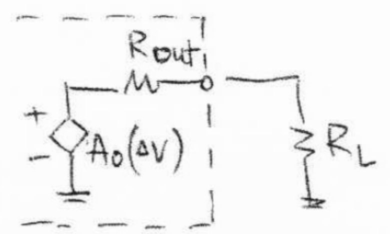
t the output voltage for the above current is

$$\pm \frac{R_2}{R_1} \times \text{input offset voltage}$$

$$= \pm \frac{2000 \times 10^3}{2 \times 10^3} \times 2\,\text{mV} = \pm 2\,\text{V}.$$

12. (10%) The unity gain buffer must be designed to drive a 100 Ω load with a gain error of 0.5%. Determine the required op amp gain if the op amp has an output resistance of 1kΩ.




Ans:

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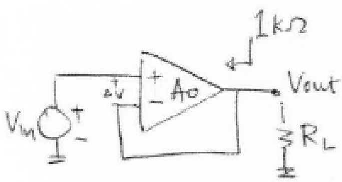
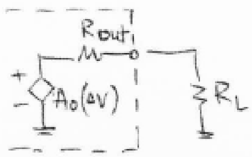



$R_L = 100\,\Omega$

Gain Error $= 0.5\%$




$$(V_{in} - V_{out}) A_0 \times \frac{R_L}{R_{out} + R_L} = V_{out}$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{1}{1 + \frac{R_{out} + R_L}{A_0 R_L}} \simeq 1 - \underbrace{\frac{R_{out} + R_L}{A_0 R_L}}_{= \epsilon}$$

$$\therefore \epsilon = \frac{R_{out} + R_L}{A_0 R_L} \Rightarrow A_0 = \frac{R_{out} + R_L}{\epsilon R_L} = \frac{1000 + 100}{0.5\% \times 100}$$

$$\approx 2200.$$