Probability midterm 2014/04/23

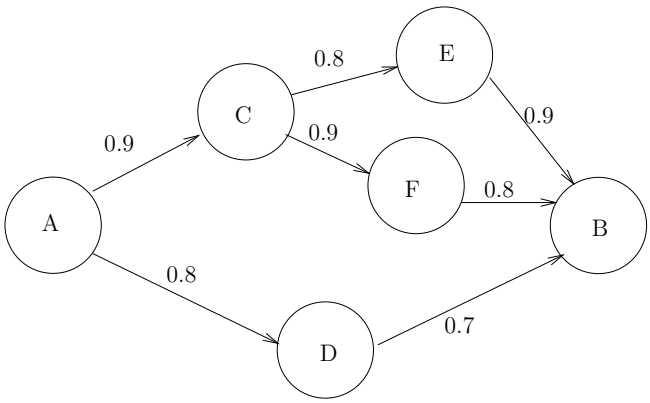
1. (10%) Romeo and Juliet have a date at a given time, and each will arrive at the meeting place with a delay between 0 and 1 hour, with all pairs of delays being equally likely. The first to arrive will wait for 20 minutes and will leave if the other has not yet arrived. What is the probability that they will meet?

2. (10%) A conservative design team A and an innovative design team B are asked to separately design a new product within a month. From past experience we know that

   - Team A is successful with probability 2/3.
   - Team B is successful with probability 1/2.
   - At least one team is successful with probability 4/5.

   Assuming that exactly one team is successful, what is the probability that it is team B?

3. (10%) A computer network connects two nodes A and B through intermediate nodes C, D, E, and F. For every pair of directly connected nodes, say i and j, there is a given probability p_{ij} (as shown in the figure) that the link from i to j is up. What is the probability that there is a path connecting A and B in which all links are up?



4. (10%) Professor May B. Right answers each of her students' questions correct with probability 1/5, independent of other questions. In each lecture, she is asked 1, 2, or 3 questions with equal probability 1/3. What is the probability that she does not answer any questions wrong in a lecture?

5. (10%) A surface is ruled with parallel lines, which are at distance 5 cm from each other. Suppose that we throw a needle of length 3 cm on the surface at random. What is the probability that the needle will intersect one of the lines?

6. (10%) The metro train arrives at a station every half hour starting at 6 a.m. You walk into the station every morning between 7:20 and 7:55 a.m., and your arrival time is a uniform random variable over this interval. What is the PDF of the time you have to wait for the first train to arrive?

7. (10%) A light bulb is known to have an exponentially distributed lifetime Y. However, the manufacturing company is experiencing quality control problems, so the parameter Λ of the PDF of Y is a random variable, uniformly distributed in the interval $[1, 2]$. We test a light bulb and record its lifetime y. What can we say about the parameter Λ?

8. (10%) Calamity Jane goes to a bank to make a withdrawal, and is equally likely to find 0 or 1 customers ahead of her. The service time of the customer ahead, if present, is exponentially distributed with parameter λ. What is the CDF of Jane's waiting time?

9. (10%) Consider 4 independent rolls of a 6-sided die. Let X be the number of 1s and Y be the number of 2s obtained. What is the joint PMF of X and Y?

10. (10%) The Celtics and the Lakers are set to play a playoff series of n basketball games, where n is odd. The Celtics have a probability p of winning any one game, independent of other games. Find the values for p for which $n = 5$ is better for the Celtics than $n = 3$.