DISCRETE MATHEMATICS

Final Examination (2013/01/14)

(**<u>You should show how to get your answers in detail or get no credit.</u>**)

1. [10%] Find the maximum length of a circuit in (a) K_8 (b) $K_{2n},\ n \in \mathbf{Z}^+$.
2. [10%] Please give two examples of planar graphs and two examples of non-planar graphs. (<u>Note that each of your four examples should contain at least 4 vertices and at least 4 edges, or you will get no credit.</u>)
3. [10%] Give an example of a connected graph that has (a) neither an Euler circuit nor a Hamilton cycle. (b) an Euler trail but no Hamilton cycle. (c) a Hamilton cycle but no Euler circuit. (d) both a Hamilton cycle and an Euler circuit. (<u>Note that each of your four examples should contain at least 5 vertices and at least 5 edges, or you will get no credit.</u>)
4. [10%] Find all the non-isomorphic complete bipartite graphs $G = (V, E)$, where $|V| = 7$.
5. [10%] Determine whether or not each of the following sets of numbers is a ring under ordinary addition and multiplication. If not, explain why.
   (a) R = the set of positive integers and zero
   (b) $R = \{a + b\sqrt{3} + c\sqrt{5} \mid a \in Z;\ b, c \in Q\}$
6. [10%] Let $(F, +, \circ)$ be a field where F is a nonempty set and "+", "$\circ$" are two binary operations on F. Please describe the eleven requirements the field must satisfy.
7. [10%] Find a simultaneous solution for the system of three congruences: (<u>Please use Chinese Remainder Theorem or get no credit.</u>)
   $X \equiv 4 \pmod{13}$, $X \equiv 5 \pmod{12}$, $X \equiv 1 \pmod{11}$
8. [10%] Prove that: A finite integral domain is a field.
9. [10%] Prove that: $\mathbf{Z}_n$ is a field if and only if n is a prime.
10. [10%] Let $G = (V, E)$ be a loop-free graph with $|V| = n \geq 2$. Prove that: If $\deg(x) + \deg(y) \geq n - 1$ for all $x, y \in V, x \neq y$, then G has a Hamilton path.