Probability   Midterm 2013/4/26

1. (20%) The taxi stand and the bus stop near Al's home are in the same location. Al goes there at a given time. If a taxi is waiting (this happens with probability 2/3) he takes it. Otherwise he waits for a taxi or a bus to come, which ever comes first. The next taxi will arrive in a time that is uniformly distributed between 0 and 10 minutes, while the next bus will arrive in exactly 5 minutes. Find the CDF of Al's waiting time, and the expected value.

2. (10%) Find the mean and variance for a continuous random variable X with the following two-sided exponential PDF

$$f_X(x) = \begin{cases} p\lambda e^{-\lambda x}, & x \geq 0, \\ (1-p)\lambda e^{\lambda x}, & x < 0, \end{cases}$$

   where $p \in (0,1)$ and $\lambda > 0$.

3. (10%) A gambler has k dollars. He makes a sequence of independent bets until he loses all money or he has $n > k$ dollars. In each bet, he wins 1 dollar with probability 1/2 and loses a dollar with probability 1/2. What is the probability that he ends up with n dollars?

4. (10%) An urns contains n balls, out of which m are red. We select k of the balls at random, without replacement. What is the probability that i of the selected balls are red?

5. (10%) Suppose that in a flip of a coin, the head faces up with probability p. The coin is flipped again and again until consecutive heads or consecutive tails show up. What is the expected value of the number of tosses?

6. (10%) X and Y are uniformly distributed in the unit square $0 \leq x, y \leq 1$. What is the joint PDF and joint CDF of X and Y?

7. (10%) Alvin's driving time to work is between 15 and 20 minutes on sunny days (with probability 2/3) and between 20 and 25 on rainy days (with probability 1/3). What is the PDF of the driving time viewed as a random variable X?

8. (10%) The annual snowfall at a particular geographic location is modeled as a normal random variable with a mean of $\mu = 60$ inches and a standard deviation of $\sigma = 20$. What is the probability that this year's snowfall will be at least 80 inches? Please express the result with $\Phi(\cdot)$, the CDF of the standard Gaussian random variable.

9. (10%) What are the means and variances of the following random variables? One must show the reasoning process for full credit.

    (a) $X \sim$ **Bernoulli**(0.2)

    (b) $G \sim$ **geometric**(0.3)

    (c) $B \sim$ **binomial**$(10, 0.6)$

    (d) $Z \sim$ **Poisson**$(2/3)$

    (e) $U_d \sim$ **uniform**$[3, 8]$

10. (12%) What are the means and variances of the following random variables? One must show the reasoning process for full credit.

    (a) $E \sim$ **exponential**(0.5)

    (b) $N \sim$ **N**$(4, 1)$

    (c) $U_c \sim$ **uniform**$(3, 8)$