[Midterm Test, 2012/11/19]

Discrete Mathematics

1. [12%] Let $\Sigma = \{x, y, z\}$. (a) How many strings in Σ^* have length 5? (b) How many strings in Σ^* have length 5 and a prefix *zyx*. (c) List all proper suffixes of string *xxyyzz*. (d) List all substrings of string *xyzxyz*.
2. [10%] Construct a state diagram for a finite state machine with $I = O = \{0, 1\}$ that recognizes all strings in the language $\{0, 1\}^*\{01\}\{0, 1\}^*\{01\}$, where I is the input alphabet and O is the output alphabet of the machine.
3. [10%] Construct a state diagram for a finite state machine with $I = O = \{0, 1\}$ that recognizes all strings with odd 1's.
4. [18%] Let B be a set with $|B| = m$. (a) How many binary relations on B are reflexive? (b) How many binary relations on B are symmetric? (c) How many binary relations on B are reflexive but not symmetric? (d) How many binary relations on B are neither reflexive nor symmetric? (e) How many binary relations on B are antisymmetric? (f) How many binary relations on B are symmetric and antisymmetric?
5. [10%] Let R be the "(exactly) divides" relation defined on $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Please draw the Hasse diagram for the poset (A, R).
6. [10%] If $A = \{1, 2, 3, 4, 5, 6\}$ and R is the equivalence relation on A that induces the partition $A = \{1, 2, 3\} \cup \{4\} \cup \{5, 6\}$, what is R?
7. [10%] Let $A = \{0, 1, 2, 3, 4, 5\} \times \{0, 1, 2, 3, 4, 5\}$, and define R on A by $(x_1, y_1)\ R\ (x_2, y_2)$ if $x_1 + y_1 = x_2 + y_2$. (a) Determine the equivalence class $[(2, 3)]$ and (b) Determine the partition of A induced by R.
8. [20%] Minimize the two finite state machines defined in Table 7.1 and Table 7.4, respectively.

Table 7.1

	ν		ω	
	0	1	0	1
s_1	s_4	s_3	0	1
s_2	s_5	s_2	1	0
s_3	s_2	s_4	0	0
s_4	s_5	s_3	0	0
s_5	s_2	s_5	1	0
s_6	s_1	s_6	1	0

Table 7.4

	ν		ω	
	0	1	0	1
s_1	s_4	s_1	0	1
s_2	s_3	s_3	1	0
s_3	s_1	s_4	1	0
s_4	s_1	s_3	0	1
s_5	s_3	s_3	1	0

[You should show how to get the answers in detail or obtain no credit.]