CALCULUS I (1011)

Midterm Exam

Department of Computer Science and Engineering
National Sun Yat-sen University

November 21, 2012, 13:20 ∼ 15:20

NAME: _____ Student ID No.: _____

Instructor: _____

General instructions:

1. Do not open this exam until you are told to begin.
2. This exam has 10 pages including this cover. There are 5 questions plus one bonus question.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each problem so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator.
7. Do NOT use pencils but black or blue ball pens for your answers.
8. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to show how you arrived at your solution.
9. Please turn off all cell phones and pagers and remove all headphones.

Problem	1	2	3	4	5	Bonus	**Total**
Points	30	40	10	10	10	15	115
Score							

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1. (30%) Find the limits in the following problems. (3% for each)

(1) $\lim_{x\to 0} \dfrac{2\tan^2 x}{x} =$ _____ .

(2) $\lim_{x\to 0} \dfrac{4(e^{2x}-1)}{e^x-1} =$ _____ .

(3) $\lim_{x\to 0} \dfrac{1-\cos x}{\sin x} =$ _____ .

(4) $\lim_{x\to 1} e^{x-1}\sin\dfrac{\pi x}{2} =$ _____ .

(5) $\lim_{x\to\infty} x\tan\dfrac{1}{x} =$ _____ .

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(6) $\lim_{x\to\infty} \dfrac{1}{2x+\sin x} =$ _____.

(7) $\lim_{x\to\infty} (2-5e^{-x}) =$ _____.

(8) $\lim_{x\to 0} \dfrac{\sin(\sin x)}{x} =$ _____.

(9) $\lim_{x\to\infty} \dfrac{x^{-1}+x^{-4}}{x^{-2}-x^{-3}} =$ _____.

(10) $\lim_{x\to 0} \dfrac{1-e^{-x}}{e^{x}-1} =$ _____.

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2. (40%) Find the derivatives of the following problems. (4% for each)

(1) $y = \dfrac{2}{\sqrt[3]{x}} + 5\cos x$, $\dfrac{dy}{dx} =$ _____ .

(2) $y = \sqrt{x}\sin x$, $\dfrac{dy}{dx} =$ _____ .

(3) $y = \dfrac{2e^x}{x^2+1}$, $\dfrac{dy}{dx} =$ _____ .

(4) $y = x\sqrt{1-x^2}$, $\dfrac{dy}{dx} =$ _____ .

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(5) $y = \log_3 \dfrac{x\sqrt{x-1}}{2}$, $\dfrac{dy}{dx} =$ _____ .

(6) $\ln xy + 5x = 30$, $\dfrac{dy}{dx} =$ _____ .

(7) $y = \sqrt{(x-1)(x-2)(x-3)}$, $\dfrac{dy}{dx} =$ _____ .

(8) $y = (x-2)^{x+1}$, $\dfrac{dy}{dx} =$ _____ .

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(9) $y = (9x^2 - 6x + 2)e^{x^3}$, $\dfrac{dy}{dx} =$ _____ .

(10) $y = \sqrt{3x + \sqrt{2 + \sqrt{1 - x}}}$, $\dfrac{dy}{dx} =$ _____ .

(Continue to the next page!)

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3. (10%) Suppose that $f(a) = g(a) = 0$, $g'(a) \neq 0$, and that f and g are differentiable on an open interval I containing a. Prove that:

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)} .$$

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4. (10%) The measurement of the edge of a cube is found to be 12 inches, with a possible error of 0.03 inch. Use differentials to approximate the maximum possible propagated error in computing (a) the volume of the cube and (b) the surface area of the cube.

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5. (10%) Suppose that $r(x) = 9x$ and $c(x) = x^3 - 6x^2 + 15x$, where x represents millions of MP3 players produced, $r(x)$ is the revenue function, and $c(x)$ is the cost function of x. Is there a production level that maximizes profit? If so, what is it?

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Bonus. (15%, bonus, no partial credit) Sketch the graph of $f(x)$. Include the coordinates of any local and absolute extreme points and inflection points as well as the asymptote:

$$f(x) = \frac{x^2 + 4}{2x} \ .$$

(End of this exam!)