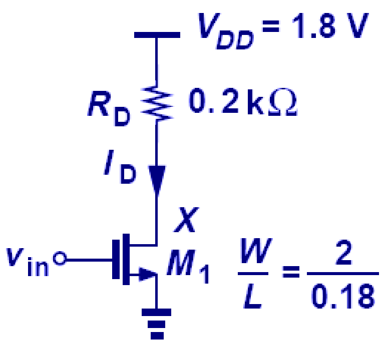
Digital Electronics Spring 2013 Final Exam 06/19/2013

Name: ID#

1. (8%) Calculate the small signal voltage gain of the CS stage if I_D=1mA, $\mu_n C_{ox}$=100uA/V^2, and V_{TH}=0.5V and λ=0. Verify that M_1=is in the saturation. $g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D}$



Ans:

$$R_D I_D < V_{DD} - (V_{GS} - V_{TH})$$

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = \sqrt{2 \times 100u \times \frac{2}{0.18} \times 1m} = \frac{1}{670.82\Omega}$$

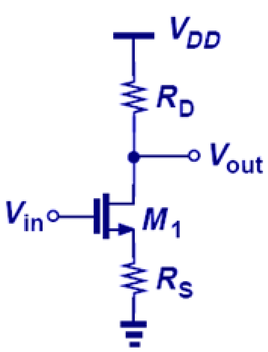
$$A_V = -g_m R_D = -\frac{1}{670.82} 0.2k = -0.298$$

$$V_{GS} = V_{TH} + \sqrt{\frac{2I_D}{\mu_n C_{ox} \frac{W}{L}}} = 0.5 + \sqrt{\frac{2 \times 1m}{100u \times \frac{2}{0.18}}} = 1.84V$$

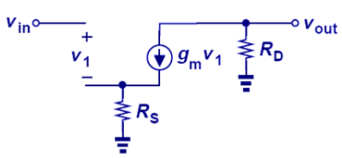
$$V_{DD} - R_D I_D = 1.8 - 0.2k \times 1mA = 1.6V \quad V_{GS} - V_{TH} = 1.84 - 0.5 = 1.34$$

the device is satuation and has a margin of 0.26V with respect to the triode region

2. (6%) Draw the small signal model of the following source degeneration circuit. And find the find the small signal gain of the following circuit.

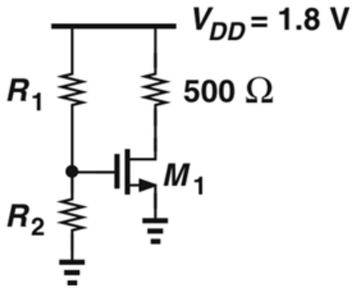


Ans:



$$v_{in} = v_1 + g_m v_1 R_S \Rightarrow v_1 = \frac{v_{in}}{1+g_m R_S} \qquad v_{out} = -g_m v_1 R_D \qquad \frac{v_{out}}{v_{in}} = -\frac{g_m R_D}{1+g_m R_S} = -\frac{R_D}{\frac{1}{g_m}+R_S}$$

3. (12%) For I_D=1mA and W/L=20/0.18. compute R_1 and R_2 such that input impedance at least 20kΩ. $\mu_n C_{ox}$=200uA/V^2, and V_{TH}=0.5V and λ=0. $I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2$



Ans:

② To get $I_{DS} = 1\,mA$,

$$\frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS} - V_{TH})^2 = 1\times10^{-3}\ A.$$

$$\frac{1}{2}(200\times10^{-6})\left(\frac{20}{0.18}\right)_1 (V_{GS} - V_{TH})^2 = 10^{-3}$$

$$(V_{GS} - V_{TH})^2 = 0.09$$

$$V_{GS} - V_{TH} = 0.3,$$

$$\text{ie. } V_{GS} = 0.7.,$$

Since $$V_{GS} = \frac{R_2}{R_1 + R_2}\times 1.8$$

$$0.7 = \frac{R_2}{R_1 + R_2}\times 1.8$$

$$0.7\ R_1 = .\ R_2,$$

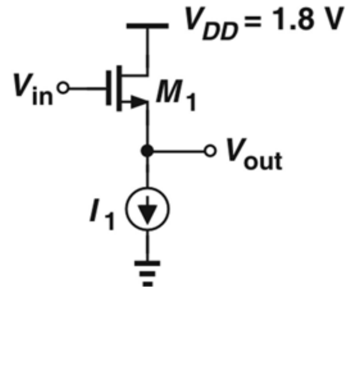
$$\therefore \frac{R_1}{R_2} = \frac{11}{7}. \qquad \text{——— ①}$$

To get input impedance $\geq 20k$.

$$R_1 // R_2 \geq 20\,k\Omega. \qquad \text{——— ②}$$

By inspection, setting $R_1 = 55\,k\Omega$ and $R_2 = 35\,k\Omega$ will satisfy both ① and ②.

4. (8%) The circuit exhibits an output impedance of less than 50Ω with a power budget of 2mW. Determine the W/L. Assume λ=0, $\mu_n C_{ox}$=200uA/V^2, and the output impedance seen at M_1 is only $1/g_{m1}$.



$$g_m = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}$$

Ans:

(53). To get $R_{out} = 50\Omega$,

$$\frac{1}{g_m} = 50\Omega$$

$$\therefore g_m = 20\,mS.$$

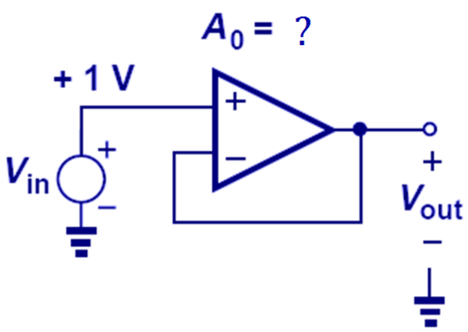
$$Power\ (P) = 1.8 \times I_{DS} = 2\times10^{-3}\,W.$$

$$\therefore I_{DS} = 1.11\,mA.$$

$$\because g_m = \sqrt{2\times(200\times10^{-6})\left(\frac{W}{L}\right)(1.11mA)}$$

$$\therefore \frac{W}{L} = 900$$

5. (6%) A unity-gain buffer shown below. V_{in+}=1V. What value of A_0 is necessary so that the output voltage is equal to 0.9999?



Ans:

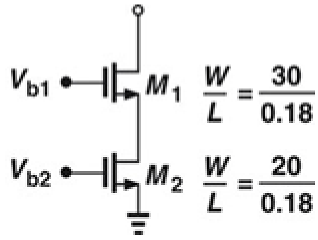
$$V_{out} = A_0(V_{in} - V_{out})\frac{V_{out}}{V_{in}} = \frac{A_0}{1+A_0} = 0.9999$$

$$\Rightarrow 0.9999 + 0.9999A_0 = A_0$$

$$\Rightarrow 0.0001A_0 = 0.9999$$

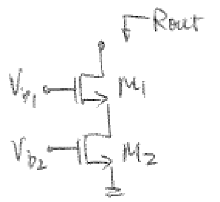
$$\Rightarrow A_0 = 9999 \approx 10000$$

6. (12%) An cascade current source as shown below must be designed for a biasing current of 0.5mA. Assume $\mu_n C_{ox}$=100uA/V^2, and V_{TH}=0.4V. (a) (8%) Neglect channel length modulation, compute the required value of V_{b2}. What is the minimum tolerable of V_{b1} if M2 bust remain saturation? (b) (4%) Assume λ=0.1V^{-1}. Calculate the output impedance of the circuit.



Ans:

14.



$\left(\frac{W}{L}\right)_1 = 30/0.18 \qquad \left(\frac{W}{L}\right)_2 = 20/0.18$

$I_{BIAS} = 0.5\ mA$

$\mu_n C_{ox} = 100\ \frac{\mu A}{V^2} \qquad V_{TH} = 0.4\ V$

(a) $I_{D2} = I_{BIAS} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 (V_{b2} - V_{TH})^2$

$$\Rightarrow V_{b2} = \sqrt{\frac{2 I_{BIAS}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2}} + V_{TH}$$

$$= \sqrt{\frac{2(0.5mA)}{(100\ \frac{\mu A}{V^2})(20/0.18)}} + 0.4V \approx 0.7\ V$$

M_2 operates in saturation as long as

$V_{GS2} - V_{TH} \le V_{DS2} \Rightarrow V_{DS2} \ge 0.3\ V.$

Observe that $V_{GS1} = V_{b1} - V_{DS2}$

$$I_{D1} = I_{BIAS} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{b1} - V_{DS2} - V_{TH})^2$$

$$\Rightarrow V_{b1} \ge \sqrt{\frac{2 I_{BIAS}}{\mu_n C_{ox}\left(\frac{W}{L}\right)_1}} + 0.4V + 0.3V$$

$$= \sqrt{\frac{2(0.5mA)}{(100\ \frac{\mu A}{V^2})(30/0.18)}} + 0.7V \approx 0.95\ V.$$

$\therefore$ Minimum $V_{b1} = 0.95V.$

(b) $R_{out} = (1 + g_{m1} r_{o2}) r_{o1} + r_{o2}$

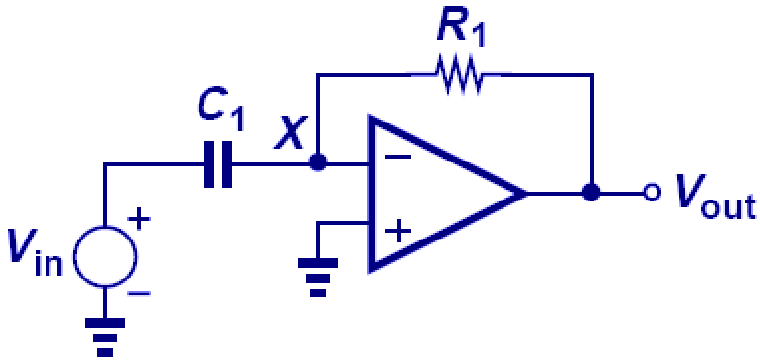
$$= \left(1 + \sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{BIAS}} \cdot \frac{1}{\lambda I_{BIAS}}\right) \cdot \frac{1}{\lambda I_{BIAS}} + \frac{1}{\lambda I_{BIAS}}$$

$$= \left[1 + \sqrt{2(100\frac{\mu A}{V^2})\left(\frac{30}{0.18}\right)(0.5mA)} \cdot \frac{1}{(0.1)(0.5m)}\right] \cdot \frac{1}{(0.1)(0.5m)}$$

$$+ \frac{1}{(0.1)(0.5mA)}$$

$$\approx 1.67\ M\Omega$$

7. (6%) A differentiator is shown below. Please find the transfer function.

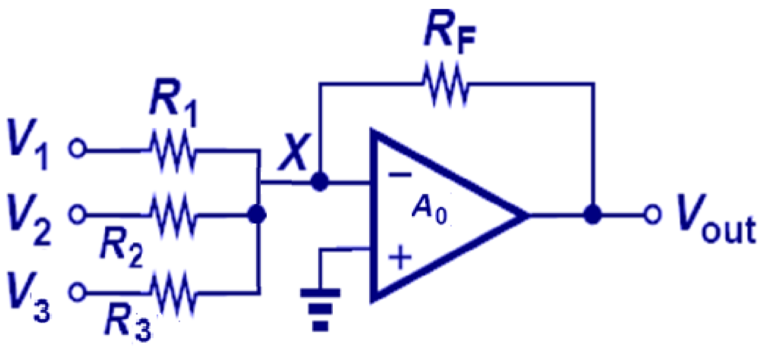


Ans:

$$\frac{V_{in}-V_X}{\frac{1}{C_1 s}}=\frac{V_X-V_{out}}{R_1} \qquad V_X=\frac{V_{out}}{-A_0}$$

$$\frac{V_{out}}{V_{in}}=\frac{-R_1C_1s}{1+\frac{1}{A_0}+\frac{R_1C_1s}{A_0}}$$

8. (6%) Find the output voltage of the following circuit in terms of V_1, V_2, V_3, R_1, R_2, R_3 and R_F.



Ans:

$$\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3}=\frac{-V_{out}}{R_F}$$

$$V_{out}=-R_F\left(\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3}\right)$$

9. (6%) An inverting amplifier must provide an input impedance of approximately 10kΩ and a nominal

gain of 4. If the op amp exhibits an open-loop gain of 1000 and output impedance of 1kΩ, determine the gain error. $\varepsilon = 1 - \frac{A_0 - \frac{R_{out}}{R_1}}{1 + \frac{R_{out}}{R_2} + A_0 + \frac{R_1}{R_2}}$

Ans:

$$\left|\frac{V_{out}}{V_{in}}\right| = \frac{R_1}{R_2} = 4 \Rightarrow R_{in} \approx R_2 = 10K\Omega$$

$$R_1 = 4R_2 = 40K\Omega$$

$$\varepsilon = 1 - \frac{A_0 - \frac{R_{out}}{R_1}}{1 + \frac{R_{out}}{R_2} + A_0 + \frac{R_1}{R_2}} = 0.51\%$$

10. (4%) The integrator of Fig. 8.51 must provide a pole at no higher than 2 Hz. If the values of R_1 and C_1 are limited to 5 kΩ and 50nF, respectively, determine the required gain of the op amp.

$$S_p = \frac{-1}{2\pi(A_0+1)RC}$$

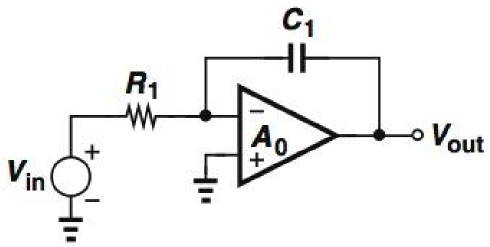


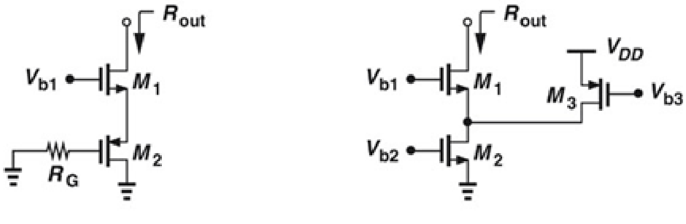
Figure 8.51

Ans:

$$S_p = \frac{-1}{2\pi(A_0+1)RC} \le -2Hz \Rightarrow 2\pi(A_0+1)(5K)(50nF) \ge 2 \Rightarrow A_0 \ge 1273$$

11. (8%) Determine the output impedance of following two circuits. Assume all transistors operate at

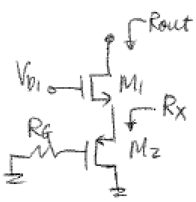
saturation and $g_m r_o >> 1$.

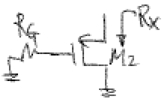


(a) (b)

Ans:

19. (a) R_X is the input impedence of a common-gate configuration:





"Looking into" the source of M_2,

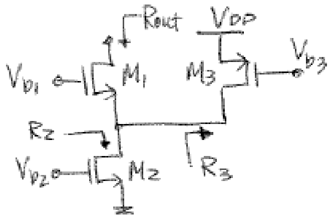
$$R_X = \frac{1}{g_{m2}} // r_{o2}$$

$$\therefore R_{out} = g_{m1} r_{o1} R_X = g_{m1} r_{o1} \left(\frac{1}{g_{m2}} // r_{o2}\right)$$

(b) From observation,

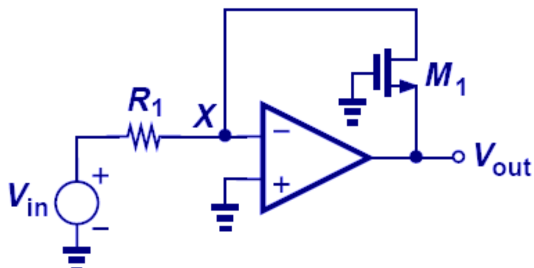
→ $R_3 = r_{o3}$ ($\because V_{SG} = 0$ in AC)

→ $R_2 = r_{o2}$ ($\because V_{SG} = 0$ in AC)



$$\therefore R_{out} = g_{m1} r_{o1} (R_2 // R_3) = g_{m1} r_{o1} (r_{o2} // r_{o3})$$

12. (6%) Please find the output voltage of the following square root amplifier.



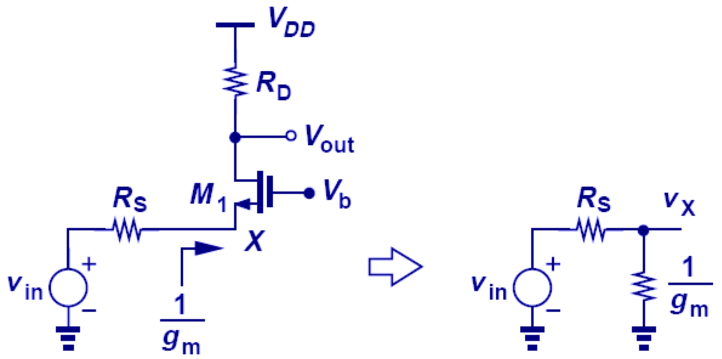
Ans:

$$\frac{V_{in}}{R_1} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})^2$$

$$V_{out} = -\sqrt{\frac{2V_{in}}{\mu_n C_{ox}\frac{W}{L}R_1}} - V_{TH}$$

$$(V_{GS} = -V_{out})$$

13. (6%) Please find the voltage gain of the following Common gate stage with source resistance.

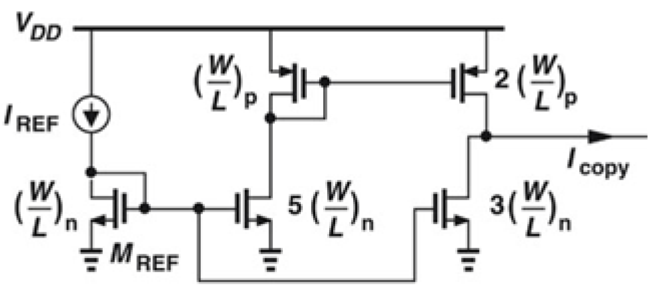


Ans:

$$v_X = \frac{\frac{1}{g_m}}{\frac{1}{g_m} + R_S} v_{in} = \frac{1}{1 + g_m R_S} v_{in}$$

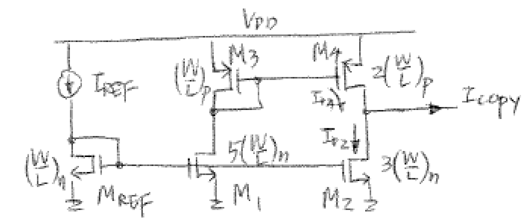
$$\frac{v_{out}}{v_{in}} = \frac{v_{out}}{v_X}\frac{v_X}{v_{in}} = \frac{g_m R_D}{1 + g_m R_S} = \frac{R_D}{\frac{1}{g_m} + R_S}$$

14. (6%) Calculate I_{copy} of the following circuit. Assume all of the transistors operate in saturation.



Ans:

(b)



$V_{GS,REF} = V_{GS,1}$: $I_{D1} = 5 I_{REF}$

$V_{GS,3} = V_{GS,4}$: $I_{D4} = 2 I_{D3} = 2 I_{D1} = 10 I_{REF}$

$V_{GS,REF} = V_{GS,2}$: $I_{D2} = 3 I_{REF}$

$\therefore I_{copy} = I_{D4} - I_{D2} = 7 I_{REF}$.