[Midterm Test, 2011/11/21]

# Discrete Mathematics

1. [10%] Let $\Sigma = \{a, b, c\}$. (a) What is $|\Sigma^4|$? (b) How many strings in $\Sigma^*$ have length at most 4?

2. [15%] Let $\Sigma = \{a, b, c\}$, and consider the string $w = abbcc$. Please list (a) all proper prefixes, (b) all suffixes, and (c) all proper substrings of $w$.

3. [10%] Construct a state diagram for a finite state machine with $I = O = \{0, 1\}$ that recognizes all strings in the language $\{0, 1\}^*\{00\} \cup \{0, 1\}^*\{11\}$, where $I$ is the input alphabet and $O$ is the output alphabet of the machine.

4. [15%] Let $A$ be a set with $|A| = n$. (a) How many binary relations on $A$? (b) How many binary relations on $A$ are reflexive? (c) How many binary relations on $A$ are symmetric? (d) How many binary relations on $A$ are reflexive but not symmetric? (e) How many binary relations on $A$ are antisymmetric?

5. [10%] Let $R$ be the "(exactly) divides" relation defined on $A = \{2, 3, 5, 6, 7, 11, 12, 35, 385\}$. Please draw the Hasse diagram for the poset $(A, R)$.

6. [10%] If $A = \{1, 2, 3, 4, 5\}$ and $R$ is the equivalence relation on $A$ that induces the partition $A = \{1, 2\} \cup \{3, 4\} \cup \{5\}$, what is $R$?

7. [10%] Let $A = \{1, 2, 3, 4, 5\} \times \{1, 2, 3, 4, 5\}$, and define $R$ on $A$ by $(x_1, y_1)\ R\ (x_2, y_2)$ if $x_1 + y_1 = x_2 + y_2$. (a) Determine the equivalence class $[(2, 4)]$ and (b) Determine the partition of $A$ induced by $R$.

8. [20%] Minimize the two finite state machines defined in Table 7.4 and Figure 7.26, respectively.

**Table 7.4**

| | $\nu$ | | $\omega$ | |
|---|---|---|---|---|
| | 0 | 1 | 0 | 1 |
| $s_1$ | $s_4$ | $s_1$ | 0 | 1 |
| $s_2$ | $s_3$ | $s_3$ | 1 | 0 |
| $s_3$ | $s_1$ | $s_4$ | 1 | 0 |
| $s_4$ | $s_1$ | $s_3$ | 0 | 1 |
| $s_5$ | $s_3$ | $s_3$ | 1 | 0 |

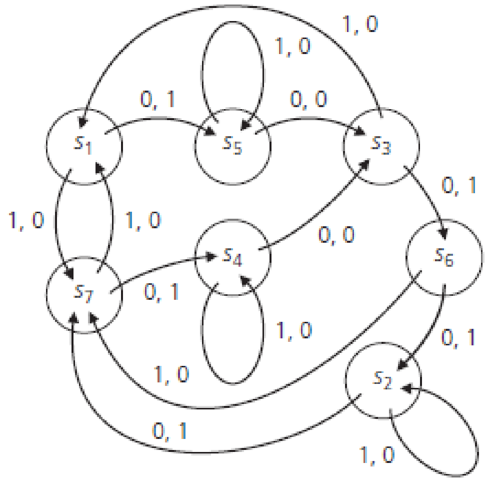

Figure 7.26

**[You should show how to get the answers in detail or obtain no credit.]**