DISCRETE MATHEMATICS

Final Examination (2012/01/09)

(**<u>You should show how to get your answers in detail or get no credit.</u>**)

1. [10%] Solve the recurrence relation: $a_{n+1} - a_n = 3n^2 - n,\ n \geq 1,\ a_0 = 3$.

2. [10%] Solve the recurrence relation: $a_{n+2} + 3a_{n+1} + 2a_n = 3^n,\ n \geq 2, a_0 = 1, a_1 = 2$.

3. [10%] Solve the recurrence relation: $a_{n+2} - 4a_{n+1} + 3a_{\mathrm{n}} = -200$, $n \geq 2$, $a_0 = 1500, a_1 = 1650$.

4. [10%] How many vertices and how many edges are there in the complete bipartite graphs $K_{8,12}$ and $K_{m,n}$, where $m, n \in \mathbf{Z}^+$?

5. [10%] Give an example of a connected graph that has (a) Neither an Euler circuit nor a Hamilton cycle. (b) An Euler circuit but no Hamilton cycle. (c) A Hamilton cycle but no Euler circuit. (d) Both a Hamilton cycle and an Euler circuit. (**<u>Note that each of your four examples should contain at least 4 vertices and at least 4 edges, or you will get no credit.</u>**)

6. [15%] Let $(R, +, \circ)$ be a ring where R is a nonempty set and "+", "$\circ$" are two binary operations on R. Please describe all of the conditions the ring must satisfy.

7. [15%] Let $(F, +, \circ)$ be a field where F is a nonempty set and "+", "$\circ$" are two binary operations on F. Please describe all of the conditions the field must satisfy.

8. [10%] (a) Find $[8]^{-1}$ in $\mathbf{Z}_{13}$. (b) Find $[16]^{-1}$ in $\mathbf{Z}_{30}$?

9. [10%] Find an integer m such that $0 < m < 23 \cdot 29 \cdot 31$ and $\begin{cases} m \equiv 0 \pmod{23} \\ m \equiv 2 \pmod{29} \\ m \equiv 3 \pmod{31} \end{cases}$ by the Chinese Remainder Theorem.