Department of Computer Science and Engineering
National Sun Yat-sen University
First Semester of 2026 PhD Qualifying Exam

Subject: Machine Learning

1. Find $\Gamma(1)$ and $\Gamma(5)$ where the Gamma function $\Gamma(x)$ is defined by

$$\Gamma(x) = \int_0^\infty u^{x-1} e^{-u} du$$

2. Let X be discrete random variable with support $\{-1, 1\}$ and probability distribution

$$p_X(x;\mu) = \left(\frac{1-\mu}{2}\right)^{(1-x)/2} \left(\frac{1+\mu}{2}\right)^{(1+x)/2} \text{ where } -1 < \mu < 1 \text{ is a parameter.}$$

Find its second moment $E(X^2)$ and entropy $H(X)$.

3. Suppose we have 3 urns L (large), M (medium), and S (small).

- L contains 3 black balls, 3 red balls, and 3 white balls
- M contains 1 black balls, 2 red balls, and 3 white balls
- S contains 2 black balls, 1 red balls, and 1 white balls

An urn is chosen with probabilities

$$p(L) = 0.3,\ p(M) = 0.5,\ p(S) = 0.2$$

and a ball is drawn at random, with each ball equally likely, from the chosen urn.

(a) What is the probability that the drawn ball is black?

(b) If the drawn ball is black, what is the probability that the chosen urn is M?

4. Consider an undirected graphical model with 4 nodes and 4 undirected edges. The incidence matrix of the graph is

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \text{ where } A_{ij} = \begin{cases} 1 & \text{if node } i \text{ is incident with edge } j \\ 0 & \text{otherwise} \end{cases}$$

Draw the graph and represent the joint probability $p(x_1, x_2, x_3, x_4)$ using clique (potential) functions, where x_i denotes the binary random variable corresponding to node i. Are x_2 and x_4 conditionally independent given x_1?

5. Consider a directed graphical model with 4 nodes and 4 directed edges. The incidence matrix of the graph is

$$B = \begin{bmatrix} -1 & -1 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \text{ where } B_{ij} = \begin{cases} -1 & \text{if edge } j \text{ leaves node } i \\ 1 & \text{if edge } j \text{ enters node } i \\ 0 & \text{otherwise} \end{cases}$$

Draw the graph and represent the joint probability $p(x_1, x_2, x_3, x_4)$ using conditional probabilities with minimum complexity, where x_i denotes the binary random variable corresponding to node i. Are x_2 and x_4 conditionally independent given x_1?