Department of Computer Science and Engineering
National Sun Yat-sen University
First Semester of 2026 PhD Qualifying Exam

Subject: **Probability**

Instructions: Answer the following questions in the answer sheet, e.g. ① 2026 . Assume uniform distribution if it is unspecified in a question.

1. Throw a fair 6-face die 5 times. What is the probability that the sum is 16?

2. Federer and Nadal play a tennis game, which finishes when one player wins 2 more points than the other. Assume each point is equally likely to be won by either player. Let G be the total number of points played to finish the game. What is $\mathbf{E}[G]$?

3. Let $X \sim \textbf{Exp}(2)$. What is $\mathbf{E}[X^2]$?

4. Let $Y \sim \mathcal{N}(0,1)$ and $Z = \Phi(Y)$ where $\Phi(\cdot)$ is the CDF of Y. What is $P\left(Z \geq \frac{1}{3}\right)$?

5. What is the expected number of flips of a fair coin until 3 consecutive heads occur?

6. Let $P \sim \textbf{Poi}(1)$ and $Q \sim \textbf{Poi}(1)$ be independent. What is the probability that $P + Q = 2$?

7. Let $N \sim \textbf{Poi}(\frac{1}{2})$. What is $\mathbf{E}[N!]$?

8. What is the probability that a random 4-member subset $S \subset \Omega = \{1, 2, \ldots, 9, 10\}$ does not contain consecutive numbers?

9. What is the probability that a random integer $1 \leq X \leq 99$ is divisible by 2 or 3 or 7?

10. An urn contains 2 pebbles, which may be both black (with probability 1/4), both white (with probability 1/4), or one black and one white (with probability 1/2). You deposit a black pebble in the urn (so there are 3 pebbles) and then randomly withdraw a pebble from the urn. Suppose the withdrawn pebble is black. What is the probability that the remaining 2 pebbles in the urn are of the same color?

Reference

$X \sim \textbf{Uni}[a,b] \;\Rightarrow\; p(x) = \dfrac{1}{b-a+1}$ for $x = a, a+1, \ldots, b$

$X \sim \textbf{Ber}(p) \;\Rightarrow\; p(x) = p^x(1-p)^{1-x}$ for $x = 0, 1$

$X \sim \textbf{Geo}(p) \;\Rightarrow\; p(x) = p(1-p)^{x-1}$ for $x = 1, 2, \ldots$

$X \sim \textbf{Poi}(\lambda) \;\Rightarrow\; p(x) = e^{-\lambda}\dfrac{\lambda^x}{x!}$ for $x = 0, 1, \ldots$

$X \sim \textbf{Bin}(n,p) \;\Rightarrow\; p(x) = \dbinom{n}{x} p^x(1-p)^{n-x}$ for $x = 0, 1, \ldots, n$

$X \sim \textbf{Uni}(a,b) \;\Rightarrow\; f(x) = \dfrac{1}{b-a}$ for $a < x < b$

$X \sim \textbf{Exp}(\lambda) \;\Rightarrow\; f(x) = \lambda e^{-\lambda x}$ for $x > 0$

$X \sim \mathcal{N}(\mu, \sigma^2) \;\Rightarrow\; f(x) = \dfrac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

$\sqrt{e} \doteq 1.65,\ \sqrt{\pi} \doteq 1.77,\ \log 2 \doteq 0.69$

$\Phi(0.5) \doteq 0.69,\ \Phi(1.0) \doteq 0.84,\ \Phi(1.5) \doteq 0.93$

$\Phi(2.0) \doteq 0.98,\ \Phi(2.5) \doteq 0.994,\ \Phi(3.0) \doteq 0.999$