

Department of Computer Science and Engineering
National Sun Yat-sen University
Second Semester of 2022 PhD Qualifying Exam
Subject: Probability

Instructions: There are 10 single-choice questions. For each question

- Making the correct choice gets 10 points
- Making a wrong choice gets -2 points
- Not making a choice gets 0 points
- Clearly show question and answer, e.g. (11) (g) or (11), in the answer sheet

1. Robert answers a question correctly with probability $2/3$, independent of any other question. In a lecture, he is asked 0, 1, or 2 questions with probability $1/2$, $1/4$, $1/4$. What is the probability that he answers at least one question incorrectly in the lecture?

- (a) $\frac{2}{9}$
- (b) $\frac{3}{8}$
- (c) $\frac{1}{8}$
- (d) $\frac{1}{3}$
- (e) $\frac{1}{4}$
- (f) none of the above

2. Let W, X, Y, Z be pairwise uncorrelated random variables with zero mean and unit variance. Define $R = \alpha W + \beta X$ and $S = \gamma X + \delta Y$. What is the correlation coefficient $\rho(R, S)$?

- (a) $\frac{\beta^2\gamma^2}{(\alpha^2+\beta^2+\gamma^2+\delta^2)^{1/2}}$
- (b) $\frac{\beta\gamma}{(\alpha^2+\beta^2+\gamma^2+\delta^2)}$
- (c) $\frac{\beta\gamma}{(\alpha^2+\beta^2)^{1/2}(\gamma^2+\delta^2)^{1/2}}$
- (d) $\frac{\beta\gamma}{(\alpha^2+\beta^2)(\gamma^2+\delta^2)}$
- (e) $\frac{\beta^2\gamma^2}{(\alpha^2+\beta^2)(\gamma^2+\delta^2)}$
- (f) none of the above

3. Pete has 2 umbrellas that he uses when commuting from home to office and back. Suppose that it rains with probability 0.5 each time he commutes. What is the steady-state probability that he gets wet by rain during a commute?
- (a) $\frac{1}{10}$
 - (b) $\frac{1}{8}$
 - (c) $\frac{1}{6}$
 - (d) $\frac{1}{5}$
 - (e) $\frac{1}{4}$
 - (f) none of the above
4. Three girls G_1, G_2, G_3 put their hats (one hat each girl) in a box and retrieve the hats (again, one hat each girl) from the box uniformly and randomly. Define random variables H_1, H_2, H_3 with $H_i = 1$ if G_i picks her own hat and $H_i = -1$ otherwise, for $i = 1, 2, 3$. Define $K = H_1 + H_2 + H_3$ and let the expectation and variance of K be denoted by m and v respectively. Which one of the following statements is true?
- (a) $m + v = 5$
 - (b) $m^2 + v = 6$
 - (c) $|m| + v = 4$
 - (d) $m - v = -5$
 - (e) $2m + v = 3$
 - (f) none of the above
5. Assume the probability of heads for a coin to be a continuous uniform random variable over $(0, 1)$. Given that the first flip results in heads, what is the probability that the next flip is heads?
- (a) $\frac{1}{2}$
 - (b) $\frac{3}{4}$
 - (c) $\frac{5}{8}$
 - (d) $\frac{3}{5}$
 - (e) $\frac{4}{7}$
 - (f) none of the above

6. Let ξ_n be the expected value of the random number of flips of a fair coin until n consecutive heads occur. For example, ξ_2 is the expected value of the random number of flips that back-to-back heads occur. Which one of the following statements is true?
- (a) $\xi_1 = 3$
 - (b) $\xi_2 = 4$
 - (c) $\xi_3 = 12$
 - (d) $\xi_4 = 30$
 - (e) $\xi_5 = 58$
 - (f) none of the above
7. An NBA finals is best-of-7 series. Assume each game is won by either team with equal probability. Let η_n be the probability that the series ends in exactly n games. Which one of the following statements is true?
- (a) $\eta_4 + \eta_7 = \eta_5 + \eta_6$
 - (b) $\eta_7 - \eta_5 = \eta_6 - \eta_4$
 - (c) $\eta_4 = 2\eta_5$
 - (d) $\eta_7 - \eta_4 = \eta_6 - \eta_5$
 - (e) $\eta_4 + \eta_5 = \eta_6 + \eta_7$
 - (f) none of the above
8. Let X and Y be continuous random variables with uniform joint PDF over the rectangle with vertices $(0, 0)$, $(1, 0)$, $(1, 2)$, $(0, 2)$ and $Z = \frac{Y}{2X}$. What is the probability that $Z > 2$?
- (a) $\frac{1}{2}$
 - (b) $\frac{1}{4}$
 - (c) $\frac{1}{6}$
 - (d) $\frac{1}{8}$
 - (e) $\frac{1}{9}$
 - (f) none of the above

9. Let X be Gaussian with mean -1 and variance 4 . Let $\omega_n = \mathbf{E}[X^n]$ be the n th moment of X . Which one of the following statements is true?
- (a) $\omega_1 + \omega_2 = 3$
 - (b) $\omega_1 + \omega_3 = -7$
 - (c) $\omega_1^2 + \omega_2 = 5$
 - (d) $\omega_3^2 - \omega_2^3 = 11$
 - (e) $\omega_1^3 - \omega_3 = 12$
 - (f) none of the above
10. A needle of length 2 cm is dropped on a surface, which is ruled with parallel lines of separation 3 cm. What is the probability that the needle intersects a line?
- (a) $\frac{1}{6\pi}$
 - (b) $\frac{\pi}{6}$
 - (c) $\frac{2}{3\pi}$
 - (d) $\frac{3}{2\pi}$
 - (e) $\frac{4}{3\pi}$
 - (f) none of the above