Department of Computer Science and Engineering
National Sun Yat-sen University
Second Semester of 2021 PhD Qualifying Exam

Subject: Probability

1. (20%) The test result of a rare disease is correct 90% of the time. A person has a probability of 10^{-4} of having the disease.

   - Given a positive test result, what is the probability of true positive?
   - Given a negative test result, what is the probability of false negative?

2. (20%) In poe divination, a.k.a. **bwa bwei**, a divination seeker drops two wooden pieces on the floor. A result is "divine" if one wooden piece is up and the other is down. We assume a result is "divine" with probability $1/2$. Let X be the number of drops until back-to-back "divines" occur.

   - Find the expectation of X
   - Find the expectation of X^2

3. (20%) A surface is ruled with parallel lines separated by 4cm. A needle of length 3cm is dropped on the surface. Find the probability that the needle does not cross any line.

4. (20%) Jane is looking for *Great Expectations*. A bookstore carries it with probability p, independent of the others. In a bookstore, Jane spends a random amount of time, exponentially distributed with parameter λ, until she either finds a copy or she determines that the bookstore does not carry it. Let the total time until Jane finds a copy be T.

   - Find the expectation of T
   - Find the variance of T

5. (20%) A gambler starts gambling with \$3. He gambles until he either accumulates \$5 or loses all his money. In each round, he either wins \$1 with probability $\frac{1}{2}$ or loses \$1 with probability $\frac{1}{2}$.

   - Find the probability that he loses all his money
   - Find the expected number of rounds for him to finish gambling