Department of Computer Science and Engineering
National Sun Yat-sen University
First Semester of 2022 PhD Qualifying Exam

Subject: Probability

1. (20%) 7 persons throw their hats in a box and then each picks one hat at random. Let H be the number of persons who pick their own hats.
   - What is the expectation of H?
   - What is the variance of H?
2. (10%) Two archers shoot at a target. The distance of each shot from the center of the target is uniformly distributed from 0 to 1, independent of the other shot. What is the probability density function of the distance of the losing shot from the center?
3. (10%) **Romeo** and **Juliet** have a date, and each will show up with a delay between 0 and 1 hour, with all pairs of delays being equally likely. The first to arrive will wait for 20 minutes and will leave if the other has not yet arrived. What is the probability that they miss each other?
4. (10%) Suppose $X \sim$ **uniform**$(0, 1)$ and $Y = X^3$. What is the probability density function of Y?
5. (10%) An absent-minded professor has 2 umbrellas that he uses when commuting from home to office and back. Suppose that it rains with probability 0.05 each time he commutes. What is the steady-state probability that he gets wet by rain during a commute?
6. (10%) Chris flips a fair coin. Let C be the random number of flips until back-to-back heads occur. What is the expected value of C?
7. (10%) John and Paul arrange schedule a dinner at 6 pm. Suppose they arrive at the restaurant with delays J and P, which are independent exponential random variables with the mean of 20 minutes. Suppose each waits for 20 minutes upon arrival. What is the probability that they meet for the dinner?
8. (10%) A surface is ruled with parallel lines separated by 3 cm. A needle of length 2 cm is dropped on the surface. Compute the probability that the needle does not cross any line.
9. (10%) A gambler starts gambling with \$3. He gambles until he either accumulates \$5 or loses all his money. In each round, he either wins \$1 with probability $\frac{1}{2}$ or loses \$1 with probability $\frac{1}{2}$. Derive the probability that he loses all his money.