Department of Computer Science and Engineering
National Sun Yat-sen University
Second Semester of 2025 PhD Qualifying Exam

Subject: Probability

Instruction: Fill out the indexed blanks in the answer sheet, e.g. (1) 2025 .

1. Throw 2 fair 6-face dice.
   - Conditional on at least one is 6, the probability of both 6s is (1).
   - Conditional on at least one is 6, the probability of even sum is (2).
2. Let $X \sim \textbf{Exp}(3)$.
$$\mathbf{E}[X] = (3)$$
$$\mathbf{E}\left[X^2\right] = (4)$$
3. Let $Y \sim \mathcal{N}(1,1)$ and $Z = F(Y)$ where $F(t) = P(Y \le t)$ is the CDF of Y.
$$P\left(Y \le \frac{1}{2}\right) = (5)$$
$$P\left(Z \le \frac{1}{2}\right) = (6)$$
4. Let $C(n)$ be the number of flips of a fair coin until the appearance of n consecutive heads.
$$\mathbf{E}[C(2)] = (7)$$
$$\mathbf{E}[C(3)] = (8)$$
5. Let P and Q be independent $\textbf{Poi}(2)$ and $S = P + Q$.
$$P(S = 2) = (9)$$
$$P(Q = 2 \,|\, S = 2) = (10)$$

Reference

$X \sim \textbf{Uni}[a,b] \;\Rightarrow\; p(x) = \dfrac{1}{b-a+1}$ for $x = a, a+1, \ldots, b$

$X \sim \textbf{Ber}(p) \;\Rightarrow\; p(x) = p^x(1-p)^{1-x}$ for $x = 0, 1$

$X \sim \textbf{Geo}(p) \;\Rightarrow\; p(x) = p(1-p)^{x-1}$ for $x = 1, 2, \ldots$

$X \sim \textbf{Poi}(\lambda) \;\Rightarrow\; p(x) = e^{-\lambda}\dfrac{\lambda^x}{x!}$ for $x = 0, 1, \ldots$

$X \sim \textbf{Bin}(n,p) \;\Rightarrow\; p(x) = \dbinom{n}{x} p^x(1-p)^{n-x}$ for $x = 0, 1, \ldots, n$

$X \sim \textbf{Uni}(a,b) \;\Rightarrow\; f(x) = \dfrac{1}{b-a}$ for $a < x < b$

$X \sim \textbf{Exp}(\lambda) \;\Rightarrow\; f(x) = \lambda e^{-\lambda x}$ for $x > 0$

$X \sim \mathcal{N}(\mu, \sigma^2) \;\Rightarrow\; f(x) = \dfrac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

$\sqrt{e} \doteq 1.65,\ \sqrt{\pi} \doteq 1.77,\ \log 2 \doteq 0.69$

$\Phi(0.5) \doteq 0.69,\ \Phi(1.0) \doteq 0.84,\ \Phi(1.5) \doteq 0.93$

$\Phi(2.0) \doteq 0.98,\ \Phi(2.5) \doteq 0.994,\ \Phi(3.0) \doteq 0.999$