

**Department of Computer Science and Engineering**  
**National Sun Yat-sen University**  
**Second Semester of 2024 PhD Qualifying Exam**

Subject : Algorithms

1. Explain each of the following terms: *NP*, *NP-complete*, *Eulerian circuit of a graph*. (15%)
2. (a) Design an algorithm for calculating the length of the *shortest path* in an undirected connected graph, where each edge has a positive length. (10%)  
(b) Analyze the time complexity of your algorithm. (5%)
3. Explain the *breadth-first* search, *depth-first* search, and *best-first* search in the tree searching strategies. What data structures are used for them? (15%)
4. (a) Explain the *2-D ranking* problem, and give an example to illustrate your answer. (5%)  
(b) Present a divide-and-conquer algorithm for solving the *2-D ranking* problem. Analyze the time complexity of your algorithm. (10%)
5. (a) Please give the definition of an AVL tree. (7%)  
(b) Describe the situation that an AVL tree increases its height after a new element is inserted. (8%)
6. Prove that the *Hamiltonian cycle* decision problem polynomially reduces to the *travelling salesperson* decision problem. (15%)
7. The logic-OR and logic-AND operations can be implemented by the linear constraints. For example, the implementation of  $w = x \text{ OR } y$  is
$$\begin{aligned}w &\geq x \\w &\geq y \\w &\leq x + y \quad (+: \text{ integer addition}) \\w, x, y &\in \{0,1\}.\end{aligned}$$
Note that in a linear constraint, you can use a multiplication of one constant and one variable, addition or subtraction. With the similar concept, please present the implementation of  $w = x \text{ AND } y$ . (10%)