

Department of Computer Science and Engineering
National Sun Yat-sen University
First Semester of 2025 PhD Qualifying Exam

Subject: Probability

Instruction: Fill out the indexed blanks in the answer sheet, e.g. ① 50 .

1. Draw 2 cards from a shuffled deck of cards.

- Given at least one drawn card is an Ace, the conditional probability that both drawn cards are Aces is ①.
- Given one drawn card is the 2 of Clubs, the conditional probability that both drawn cards are Clubs is ②.

2. Let $Z \sim \mathcal{N}(1, 2)$.

$$\mathbf{E}[Z^2] = \underline{\textcircled{3}}$$

$$\mathbf{E}[Z^3] = \underline{\textcircled{4}}$$

3. $X \sim \mathbf{Exp}(3)$ and $Y = F(X)$ where $F(x) = P(X \leq x)$.

$$P\left(X \leq \frac{1}{3}\right) = \underline{\textcircled{5}}$$

$$P\left(Y \leq \frac{1}{3}\right) = \underline{\textcircled{6}}$$

4. 3 persons mix their hats in a box and each picks one hat randomly. Let X be the number of person(s) *not* picking own hat(s).

$$\mathbf{E}[X] = \underline{\textcircled{7}}$$

$$\mathbf{E}[X^2] = \underline{\textcircled{8}}$$

5. Let Q and R be independent $\mathbf{Poi}(1)$ and $S = Q + R$.

$$P(S = 1) = \underline{\textcircled{9}}$$

$$P(Q = 1|S = 3) = \underline{\textcircled{10}}$$

Reference

$$X \sim \mathbf{Uni}[a, b] \Rightarrow p(x) = \frac{1}{b-a+1} \text{ for } x = a, a+1, \dots, b$$

$$X \sim \mathbf{Ber}(p) \Rightarrow p(x) = p^x(1-p)^{1-x} \text{ for } x = 0, 1$$

$$X \sim \mathbf{Geo}(p) \Rightarrow p(x) = p(1-p)^{x-1} \text{ for } x = 1, 2, \dots$$

$$X \sim \mathbf{Poi}(\lambda) \Rightarrow p(x) = e^{-\lambda} \frac{\lambda^x}{x!} \text{ for } x = 0, 1, \dots$$

$$X \sim \mathbf{Bin}(n, p) \Rightarrow p(x) = \binom{n}{x} p^x (1-p)^{n-x} \text{ for } x = 0, 1, \dots, n$$

$$X \sim \mathbf{Uni}(a, b) \Rightarrow f(x) = \frac{1}{b-a} \text{ for } a < x < b$$

$$X \sim \mathbf{Exp}(\lambda) \Rightarrow f(x) = \lambda e^{-\lambda x} \text{ for } x > 0$$

$$X \sim \mathcal{N}(\mu, \sigma^2) \Rightarrow f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\sqrt{e} \doteq 1.65, \sqrt{\pi} \doteq 1.77, \log 2 \doteq 0.69$$

$$\Phi(0.5) \doteq 0.69, \Phi(1.0) \doteq 0.84, \Phi(1.5) \doteq 0.93$$

$$\Phi(2.0) \doteq 0.98, \Phi(2.5) \doteq 0.994, \Phi(3.0) \doteq 0.999$$