

**Department of Computer Science and Engineering**  
**National Sun Yat-sen University**  
**First Semester of 2025 PhD Qualifying Exam**

Subject : Algorithms

1. Please explain P, NP-hard, NP-complete problems as well as reduction. Then, prove that the vertex-cover problem is NP-complete. (20%)
2. Please use the master method to obtain the asymptotic “ $\theta$ ” bounds of the following cases (15%):
  - (a)  $T(n) = 4T(n/2) + n$
  - (b)  $T(n) = 9T(4n/6) + 1$
  - (c)  $T(n) = 4T(n/5) + n \lg n$
3. Please design an algorithm to find the minimum and maximum values in a set of  $n$  elements using  $3\lfloor n/2 \rfloor$  comparisons in total. (5%)
4. Please use the dynamic programming method to solve the matrix-chain multiplication and provide a simple example to illustrate how it works. (20%)
5. Please explain how the Bellman-Ford algorithm solves the single-source shortest-paths problem in the general case where edge weights may be “negative” and provide a simple example to illustrate it. (10%)
6. Please explain why any comparison sort algorithm requires  $\Omega(n \lg n)$  comparisons in the worst case. (10%)
7. Please explain the minimum spanning tree (MST) and how to solve it using the Kruskal’s and Prim’s algorithms. (20%)