Department of Computer Science and Engineering
National Sun Yat-sen University
First Semester of 2024 PhD Qualifying Exam

Subject: <u>Probability</u>

1. Consider a class of 20 students. Assume the birthday of each student is uniformly distributed over 365 calendar days, independent of other students.

   (a) (10%) What is the expected value of the total number of different birthdays?

   (b) (10%) What is the expected value of the total number of same-birthday student pairs?

2. Draw 2 cards in sequence without replacement from a deck of 52 cards. Let A be the event that the first drawn card is an Ace, and B be the event that the second drawn card is a black-suit (i.e. Spades or Clubs) card.

   (a) (10%) What is the conditional probability $P(A|B)$?

   (b) (10%) What is the conditional probability $P(B|A)$?

3. A family has 2 children. Find the conditional probability that both children are girls in light of the following information, respectively.

   (a) (10%) At least one child is born on Sunday.

   (b) (10%) At least one child is a girl and born on Sunday.

4. Consider a fair coin and a biased coin, which lands Heads with probability 2/3. You select one of the coins at random with equal probability and flip the chosen coin twice. It lands Heads both times.

   (a) (10%) What is the probability that the chosen coin is the fair coin?

   (b) (10%) What is the probability that the next toss of the chosen coin is Heads?

5. You flip a fair coin until back-to-back Heads occurs for the first time. Then you stop.

   (a) (10%) What is the expected value of the number of tosses?

   (b) (10%) What is the variance of the number of tosses?