Department of Computer Science and Engineering
National Sun Yat-sen University
First Semester of 2023 PhD Qualifying Exam

Subject: Probability

There are 5 *single-choice questions in this exam. Each question is scored by the following rules.*

- 20 *points for the correct answer*
- -4 *points for a wrong answer*
- 0 *points for not answering the question*

1. Assume for a coin that the prior probability of heads is a continuous uniform random variable over $(0, 1)$. We observe that the first 2 flips of the coin are both heads. What is the probability that the next flip of the coin is heads?

   (a) $\frac{1}{3}$
   (b) $\frac{1}{2}$
   (c) $\frac{2}{3}$
   (d) $\frac{3}{4}$
   (e) $\frac{4}{5}$
   (f) none of the above

2. Guy answers a question correctly with probability 1/4, independent of any other question. In a lecture, he is asked 0, 1, or 2 questions with probability 1/6, 1/2, 1/3. What is the probability that he answers at least one question incorrectly in the lecture?

   (a) $\frac{5}{8}$
   (b) $\frac{11}{16}$
   (c) $\frac{3}{4}$
   (d) $\frac{13}{16}$
   (e) $\frac{7}{8}$
   (f) none of the above

3. Girls G_1, G_2, G_3, G_4 put their hats (one hat each girl) in a box and retrieve the hats (again, one hat each girl) from the box uniformly and randomly. Define random variables H_1, H_2, H_3, H_4 with $H_i = 1$ if the girl G_i retrieves her own hat and $H_i = 0$ otherwise. Let $K = H_1 + H_2 + H_3 + H_4$ and the mean and variance of K be m and v respectively. Which one of the following statements is true?

   (a) $v - m = 1$
   (b) $m + v = 2$
   (c) $m^2 + v^2 = 3$
   (d) $(m + 2v)^2 = 4$
   (e) $m^2 + v = 5$
   (f) none of the above

4. Let W, X, Y, Z be pairwise uncorrelated random variables with zero mean and unit variance. Define $R = \alpha W + \beta X + \gamma Y$ and $S = \kappa X + \mu Y + \nu Z$. What is the correlation coefficient $\rho(R, S)$?

   (a) $\frac{\beta\kappa+\gamma\mu}{(\alpha^2+\beta^2+\gamma^2)^{1/2}(\kappa^2+\mu^2+\nu^2)^{1/2}}$
   (b) $\frac{\alpha\kappa+\beta\mu+\gamma\nu}{(\alpha^2+\beta^2+\gamma^2+\kappa^2+\mu^2+\nu^2)}$
   (c) $\frac{\beta\kappa+\gamma\mu}{(\alpha^2+\beta^2+\gamma^2+\kappa^2+\mu^2+\nu^2)^{1/2}}$
   (d) $\frac{\alpha\kappa+\beta\mu+\gamma\nu}{(\alpha^2+\beta^2+\gamma^2)(\kappa^2+\mu^2+\nu^2)}$
   (e) $\frac{\beta\kappa+\gamma\mu}{(\alpha+\beta+\gamma+\kappa+\mu+\nu)^{1/2}}$
   (f) none of the above

5. Hans has 2 umbrellas to use, when it is raining, for the commutes from home to office or from office to home. Suppose that it rains with probability $1/3$ each time he commutes. What is the steady-state probability that it rains and he is without an umbrella during a commute?

   (a) $\frac{1}{15}$
   (b) $\frac{1}{12}$
   (c) $\frac{1}{11}$
   (d) $\frac{1}{10}$
   (e) $\frac{1}{9}$
   (f) none of the above